# Controlling Wild Bodies Using Linear Temporal Logic

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## Motivation

We control a group of bodies from specifications of tasks given in a high-level, human-like language without dynamical system modeling, precise state estimation or state feedback.

## Linear Temporal Logic (LTL)

*disjunction* ($\vee$), *negation* ($\neg$) *conjunction* ($\wedge$) *implication* ($\Rightarrow$), *equivalence* ($\Leftrightarrow$), *eventually* ($\diamondsuit$), and *always* ($\square$).
Task specifications (Kress-Gazit, et al., 2005):

- Navigation: $\diamondsuit\pi_1$
- Sequencing: $\diamondsuit(\pi_1 \wedge \diamondsuit(\pi_2 \wedge \diamondsuit\pi_3)$
- Coverage: $\diamondsuit\pi_1 \wedge \diamondsuit\pi_2 \wedge \cdots \wedge \diamondsuit\pi_k$
- Avoiding regions: $\neg(\pi_1 \vee \pi_2 \cdots \vee \pi_k)\mathcal{U}\pi_{final}$
- Patrolling: $\square(\diamondsuit\pi_1 \wedge \diamondsuit\pi_2 \wedge \ldots \diamondsuit\pi_k)$.

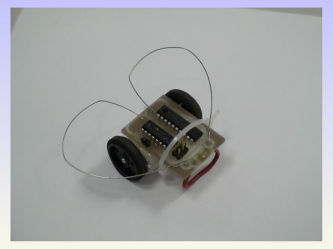
## Related work

- LTL for robot control: Kress-Gazit, Fainekos, Pappas, 2005, 2007, 2009; Loizou, Kyriakopolous, 2005; Kloetzer and Belta, 2007; Fainekos, 2011; Wu et al. 2009; Finucane, Kress-Gazit, 2010 ; Bhatia, Karvaki, Vardi, 2010.
- Nonprehensile manipulation: Erdmann, Mason, 1988; Goldberg 1993; Bohringer, Bhatt, Donald, Goldberg, 2000; Reznick, Moshkoich, Canny, 2000. Vose, Umbanhowar, Lynch, 2011.
- Virtual fences for herding: Butler, Corke, Peterson, Rus, 2004.
- Fire evacuation strategies: Chalmet, Francis, Saunders, 1982.
- Dynamical Billiards and Ergodicity

## Wildness conditions

Exploit a high-level property:For any region $r \in R$, $b$ moves on a trajectory that causes it to repeatedly strike every open interval in $\partial r$ (the boundary of $r$), with non-zero, non-tangential velocities.

## Our wild vehicles

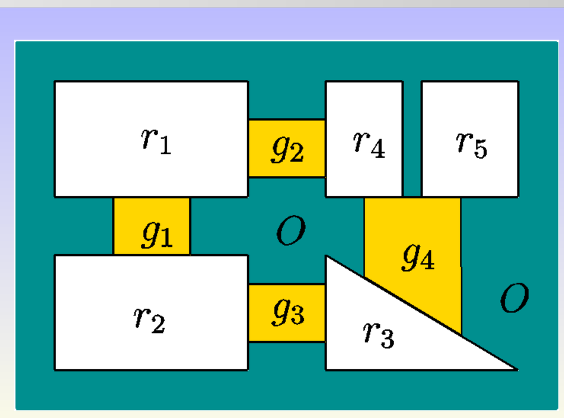

## Notation

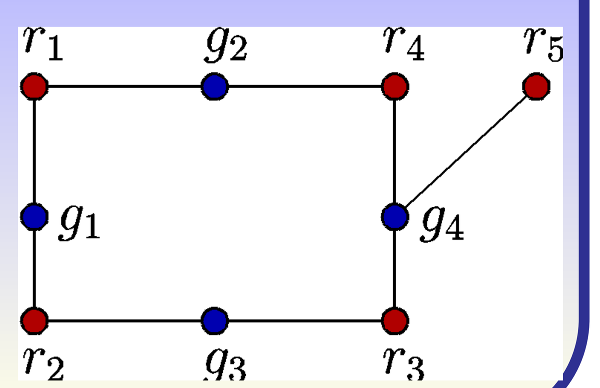
- $R$ regions
- $G$ gates
- $M(g)$ gate mode for $g \in G$
- $X$ body state space.
- $Z = M \times X$ hybrid state space

The mode $m \in M(g)$ could allow one of four behaviors:

1. No passage between $r$ and $r'$
2. Passage only from $r$ to $r'$
3. Passage only from $r'$ to $r$
4. Bidirectional passage between $r'$ and $r$

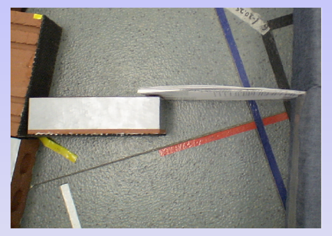
## Regions and gates



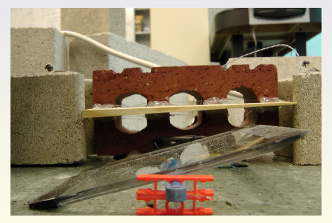
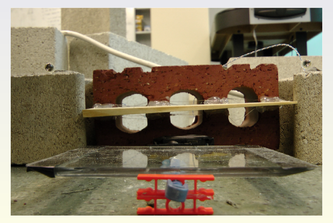
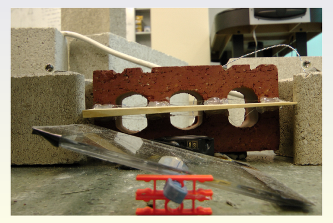



## Gates

Static




Controllable

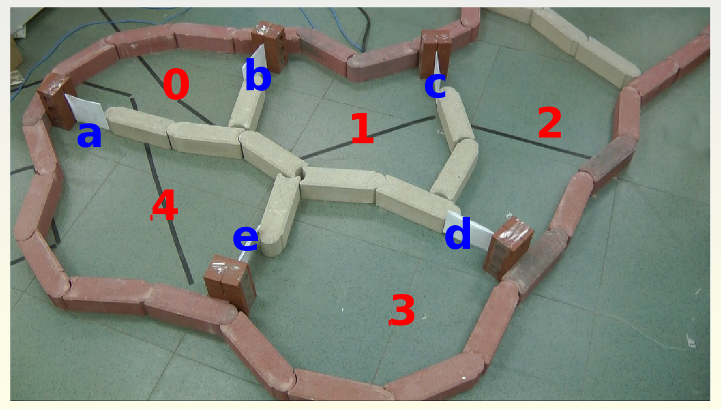
## Controlling one wild body

- $Q = M \times R$ discrete abstraction
- $D_1 = (Q, q_0, \rightarrow_1)$ discrete transition system
- $q_0 = (m_0, r_0)$ initial composite mode
- $q \rightarrow_1 q'$ if and only if $q = (m, r)$ and $q' = (m', r')$

We used a model checker to find $\tilde{q} = (q_0, q_1, \ldots)$ for $D_1$ satisfying $\tilde{q} \models \phi$.
Example :

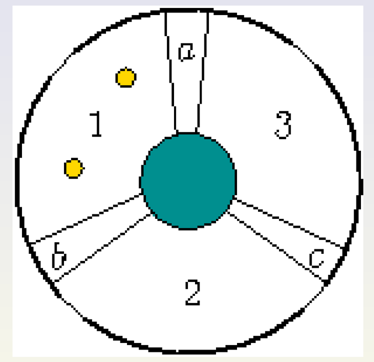
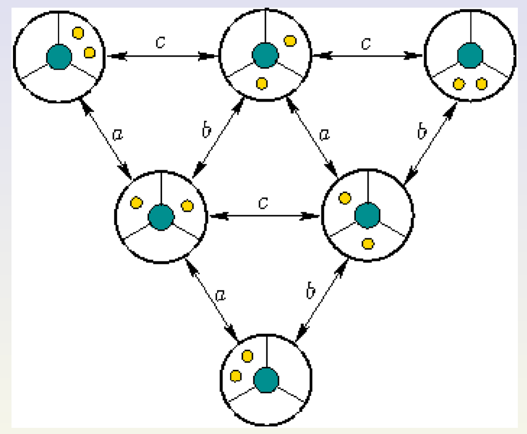
$\phi = \diamondsuit(\pi_2 \wedge \diamondsuit(\pi_1 \wedge \diamondsuit(\pi_0 \wedge \diamondsuit\pi_4)))$

1. Set gate $c$ to allow passage from $r_2$ to $r_1$.
2. Set gate $b$ to allow passage from $r_1$ to $r_0$.
3. Set gate $a$ to allow passage from $r_0$ to $r_4$.




## Controlling multiple wild bodies

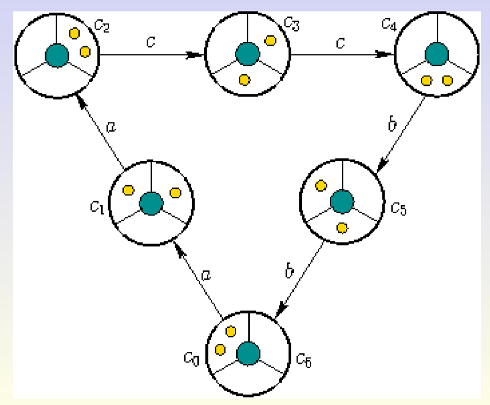
- $n$ identical, indistinguishable bodies
- $C$ set of possible body positions
- $Q_n = M \times C$ discrete abstraction.
- $D_n = (Q_n, q_0, \rightarrow_n)$ discrete transition system
- $q_0 = (m_0, c_0)$
- $q \rightarrow_n q'$ if and only if $q = (m, c)$ and $q' = (m', c')$

A model checker is used to find $\tilde{q} = (q_0, q_1, \ldots, q_k)$ for $D_n$ that satisfies $\tilde{q} \models \phi$.

## A simple example

Both bodies are initially in $r_1$ bring them to $r_3$, then $r_2$, and then return to $r_1$
$\diamondsuit(\pi_1 \wedge \diamondsuit(\pi_3 \wedge \diamondsuit(\pi_2 \wedge \diamondsuit\pi_1)))$



## Future work

- Currently considering other gates and sensor designs.
- We are enhancing the bodies with sensing, controllable actuation, and computation.
- Need to analize time to enter the gate for various motion models and region shapes.