Assembly of Hinged Polygon Triangulations

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Abstract— **In this paper we consider the problem of assembly a hinged dissection of a polygon using a mechanical gripper. We proposed some conjectures about the stability positions in which the gripper will stop and some conditions in which the mechanical gripper will not be able to reassemble the original polygon. Experimental results support the conjectures. Future directions are outlined and alternative approaches for solving the problem are considered.**

I. INTRODUCTION AND RELATED WORK

The use of mechanical grippers has been widely considered in the literature in the context of parts orienting and assembly [12].

The most closely related work, is the work of Goldberg [7] [15], where an algorithm to orient a polygon was proposed, an extension to piecewise algebraic curves was later presented in [14]. Another related line of work due Erdmann in the context of tray tiling [3] and preimage backchaining in compliant motion planning [11] [10]

In another direction, the problem of hinged dissections [9] has been considered in [6] [5]. In this problem, the central idea is to shape one polygon into another using a finite number of hinges that can be place arbitrarily in the original polygon. In this problem, however, there are no motion planning considerations.

Closely related also is the concepts of triangulation, a fundamental problem in computational geometry [16] [8], which consists in the decomposition of a polygon into a set of triangles with the property that the partition into non-overlapping triangles whose union is the polygon.

Related works in the literature are assembly of pieces in manufacturing [12], the problem of self assembly [4] and robotic origami folding [1] as well as distributed manipulation systems [16]

II. PROBLEM STATEMENT

Given a list of n vertices describing a polygon, find, if possible a sequence of squeeze actions guaranteed to assemble a hinged-radial-triangulation of the polygon as illustrated in the figure 1. More precisely, we will take a convex polygon, find its center of mass and connect lines from its center of mass to all its vertices to generate a radial triangulation. We will put hinges in all the corners but one 1 to obtain a chain of triangles. The objective is to find out if it is possible to assemble the polygon for any configuration (up to symmetry) only by squeezing motions of a mechanical gripper.

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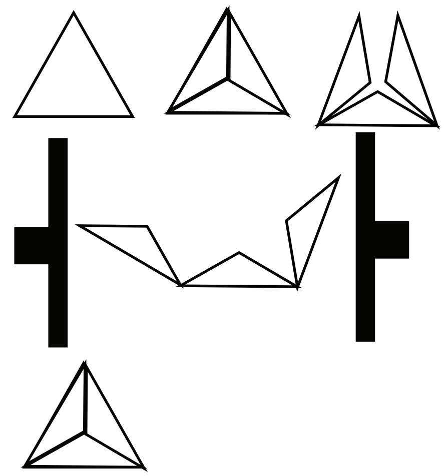

Fig. 1. Problem Statement

We are going to have $n-1$ joints. Let A_i denote the possible angles of the ith joint, we know that each $A_i \subset [0, 2\pi)$. Therefore the configuration space of the system can be expressed as: $Q = \prod A_i$

The definition of the problem is as follows I will have an initial point in my configuration space q_o and this point maybe unknown, and get it to through a set of squeezing actions $u_1, u_2, ..., u_n$ $u_i \in [0, 2\pi)$ to a final desired configuration q_f

The problem is hard for several reasons. First, the problem may be considered an instance of the general motion planning problem which is NP-hard, since we are going from a known configuration q_o to a final desired configuration q_f while avoiding a set of obstacles.

Second, the possibility of doing planning with uncertainty will be considered, since and the configuration of the chain of triangles my not be precisely known, and yet we would like to drive the triangle chain, if possible, to a point (or a set of points) in the configuration space.

Third, we will not have control over all the joints of the triangular chain, somehow the system will be underactuated since we will only have control over the actions of the gripper to modify the joint angles of the chain.

In the following sections, we will try to argue that the properties of the system such as stability, contacts among others may give a way to solve the problem in some cases.

A. Mechanical Assumptions

We will consider the following assumptions, these assumptions are similar to those made in [13][2] with the exception of the last 3 assumptions that are specific for the problem at hand since we are dealing with more than one body.

1) All motions in plane and inertial forces are negligible
2) Parallel gripper jaws
3) The jaws make contact simultaneously
4) The direction of gripper motion is orthogonal to the jaws.
5) Jaws close until the closing deform the chain of polygons
6) The jaws can be opened to the maximum length of the chain
7) The size of the hinges is negligible
8) There is zero friction between the part and the jaws.
9) There is no elasticity that makes the objects bounce. The collisions are inelastic

III. RESULTS

In this section, we will present some conjectures and some initial exploration to solve the problem of re-assembling the hinge dissection of a polygon.

Conjecture 1. If any of the angles of the hinges is $\alpha_i \geq \pi$ no sequence of squeezing operations will be able to reassemble the polygon. This case is illustrated in the figure . If any of the angles is less than π I will not be able to reassemble the polygon since at best after a squeezing operation we will have the fully extended polygonal chain where all the angles are equal to π and it will not be possible to assemble the polygon. T

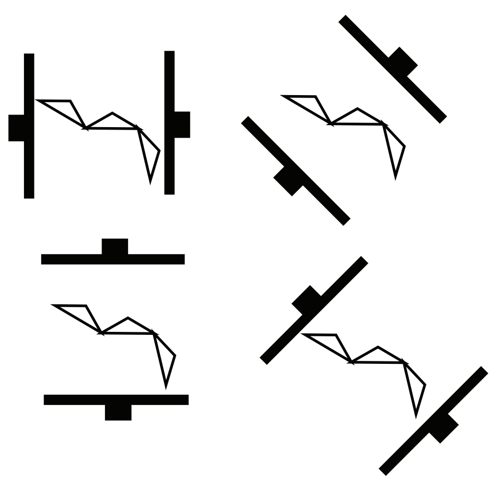

Fig. 2. Illustration of the first conjecture.

From this point on, we will assume that the angles of the are less than π.

proof:

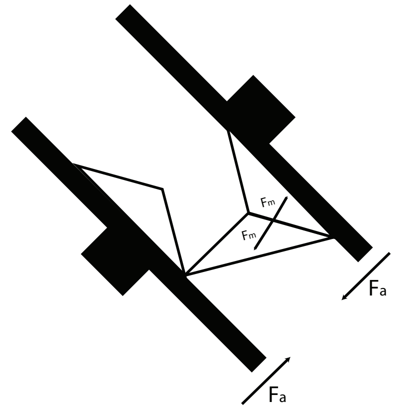

Fig. 3. Illustration of the second conjecture. Two edges come to contact

Conjecture 2. Any pushing action will take a point in the configuration space to a equilibrium configuration (sum of forces is zero and the sum of momentum is zero). This can happen because either two edges will come to contact or the gripper and further pushing will break the piece apart (this case is illustrated in the figure) or the gripper finds two or more vertices perfectly aligned and will be pushing orthogonal to the direction of rotation (as illustrated in the figure).

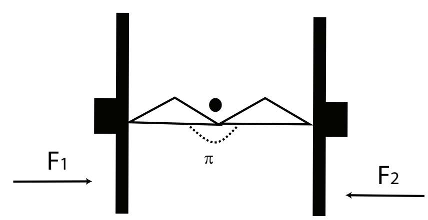

Fig. 4. Illustration of the second conjecture. Pushing orthogonal to the direction of rotation

Conjecture 3. Whenever two inner edges come into contact the for some range of directions in which the force may be applied, the system can be considered a single body. This means that we may be able to find some actions that will not separate two inner edges once they are in contact. We illustrate this idea in the figures .

Conjecture 4. If all the angles of the hinges are $\alpha_i < \pi$ there is a sequence of squeezing movements that will reassemble the polygon.

It follows from conjecture 1, 2 and 3. A sample planner and the pushing direction for the gripper are illustrated in the figures and .

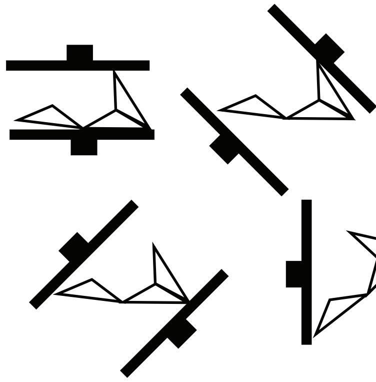

Fig. 5. Illustration of the third conjecture (a): For some directions two triangles can be considered a single rigid body

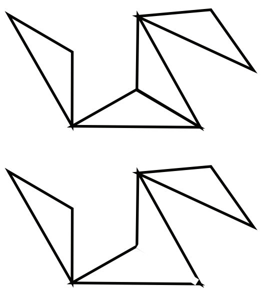

Fig. 6. Illustration of the third conjecture (b): For some directions two triangles can be considered a single rigid body

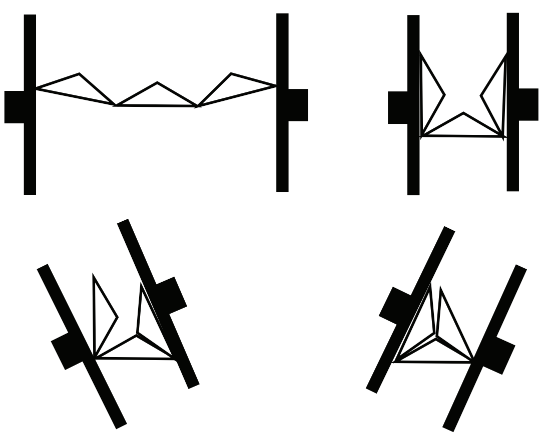

Fig. 7. A sample planner

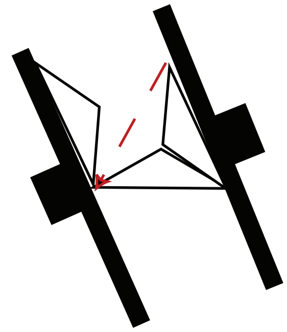

Fig. 8. Pushing Direction

IV. EXPERIMENTS

All the experiments were carried out in Working Model 2D. We implemented in a realistic physics simulator the system, consisting on the parallel grippers and a hinged dissection of different polygons. Experimental results in a physics simulator support most of the conjectures. Experiments were carried out to test the stability of the final configuration, the quasi-static assumption, the proposed planners, and the conjectures. A snapshot of the simulator can be seen in the figure 9.

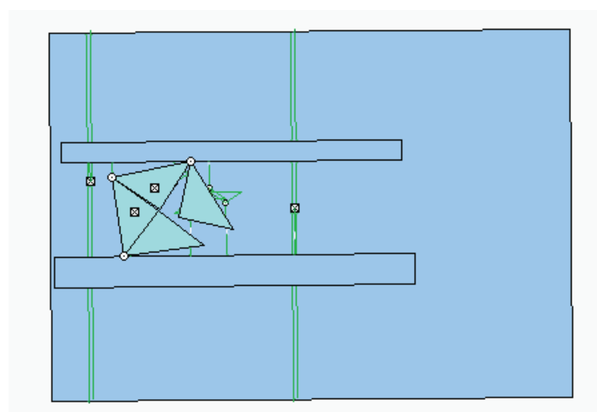

Fig. 9. Simulation of the gripper

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

In this paper we present the problem of assembly a hinged dissection using a parallel gripper. We mentioned some of the reasons that make the problem hard. We proposed some conjectures about the stability conditions in which the gripper will stop and presented a condition in which the mechanical gripper will not be able to solve the problem.

We implemented in a realistic physics simulator the system, consisting on the parallel grippers and a hinged dissection of different polygons. Experimental results in a physics simulator support the conjectures.

The assembly of a hinged dissection of a polygon is an interesting problem but is hard due to the need to model the forces and interactions of the chain of polygons. This has undoubtedly, complicated the mechanical analysis of the problem and that should be solved in order to find a planner.

B. Future Work

There are several avenues of work to continue the work presented in this paper. Among others we would like to analyze the following problems:

1) Monotonicity of the Planner: The first one is the following conjecture that will try to find ”monotonicity” in the planner:

Conjecture: Let us call Q_k the possible set of configurations of the hinges at stage k. Let us call $U_k \subset [0, 2\pi)$ a subset of actions of the gripper. For any stage k, it is possible to choose a U_k and some measure μ such that $Q_{k+1} = f(Q_k, U_k)$ and $\mu(Q_k) \leq \mu(Q_{k+1})$

This will imply that during the execution of the planner, we will be at each step guaranteeing that the uncertainty in the configuration of the chain of polygons will only decrease because we will be able to find an action that will not break the fact that two pieces are connected and will attempt to connect two other inner edges. This is illustrated in the following figure.

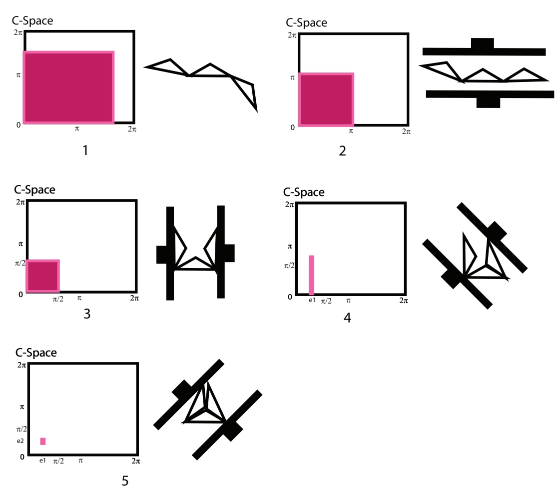

Fig. 10. Monotonicity of the plan

2) Cell decomposition of the configuration space: **Conjecture:** Combinatorial changes in the convex hull of the hinged chain of triangles induces a cell decompomposition of the C-space $Q = \cup_{i=1}^{m} Q_i$. We know in which of these regions is the goal configuration. A planner should be explored such that $\pi : Q_i \rightarrow U$. This idea is illustrated in the figure 11.

Sketch of the proof: The number of combinatorial changes of the convex hull is finite

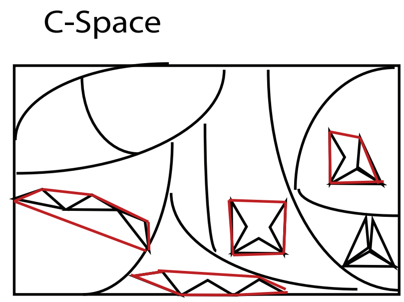

Fig. 11. Cell decomposition of the configuration space

3) Independent gripper planner: If we assume that we can independently manipulate each jaw of the gripper and that we can stop the motion when two inner edges come into contact, we can propose the following

Conjecture: There is a $\theta(n)$ steps greedy planner that uses two independent pushers that is able to assemble the polygon under most configurations

Proof: We first push orthogonal to the axis of the chain to obtain a configuration with all the external angles equal to π. Then, we will put the “left gripper” orthogonal to the left most vertex and push the rightmost vertex with the “right gripper” in a proper angle until no further force can be applied (two inner edges will be in contact). The left gripper is acting like and anchor to stabilize and prevent motions from the chain of triangles. We will alternate the orthogonal supporting gripper with the the pusher gripper and execute $n - 1$ pushing actions with each jaw. We illustrate this idea in the figure 12.

4) Solvability of the Kinematic Problem: It will be interesting to considering the kinematic only problem. The solution to this problem might not be not trivial since some triangles will serve as obstacles in the configuration space an this will change during the execution of the planner. There is some instances, however, where the problem is solvable, as an example we have the following idea:

Conjecture: For a circumscribed polygon the planning problem can be solved if the external angles of the hinges are π.

Proof: We know that the internal angles are $\alpha_1 = \frac{(\pi - \frac{2\pi}{n})}{2}$

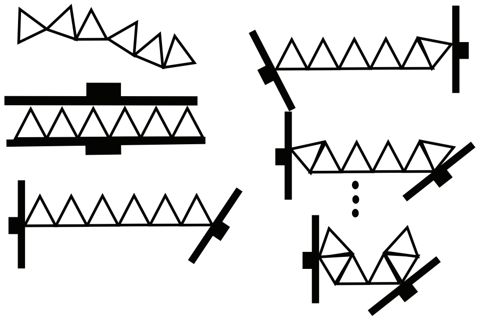

Fig. 12. Sketch of a planner for the independent gripper case

and the angle between the triangles is $\alpha_2 = \pi - 2 \bullet \alpha_1$. A motion planner will move each joint α_2 to solve the problem.

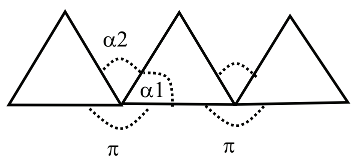

Fig. 13. A simple solution for the kinematic version of the problem

5) Stability of the Final Solutions: for some range of directions in which the force may be applied, the system can be considered a single rigid body In general most of the actions will not disrupt the final configuration. However, some actions of the parallel gripper may break it.

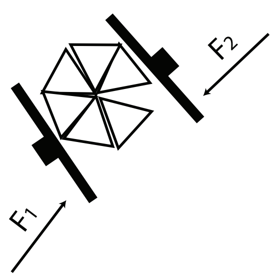

Fig. 14. Stability of the final solution

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