1. [10 points] Consider a variant of a PDA, called a queue-machine, which is an NFA with a queue instead of a stack. A queue is a data structure which allows reading and popping the symbol at the front of the queue, or pushing a symbol to the end of the queue (i.e., a first in, first out model). A queue-machine may not always halt for some inputs. Give a formal definition of what a queue machine is, and then argue that for every TM, there is a queue-machine that simulates it.

2. [10 points] Let \( \log^*(x) = n \), for \( x, n \in \mathbb{N} \), be the number of times \( \log \) should be successively applied to \( x \) to obtain 0. That is:

\[
\underbrace{\log(\log(\cdots \log(\log(x) \cdots)))}_{\log^*(x)=n\text{times}} = 0.
\]

Describe a Turing Machine that computes \( \log^*(x) \). Assume that the input to the Turing Machine is a string of the form \( 1^x \).

3. [10 points] Prove that the class of recognizable languages is closed under Kleene *. (Hint: Think nondeterministically.)

4. [10 points] For this problem, consider the Turing Machine model in which the machine can move its head to the left, to the right, or to stay in the same tape cell. Consider the following language:

\[
A = \{ \langle M \# w \rangle \mid M \text{ moves its head to the left at least once on input } w \}
\]

Is \( A \) recognizable? Is \( A \) decidable?