

- Give a counterexample to show that the following construction fails to prove the class of context-free languages is closed under star: Let A be a CFL that is generated by the CFG $G = (V, \Sigma, R, S)$. Add the new rule $S \rightarrow SS$ and call the resulting grammar G' . This grammar is supposed to generate A^* .
- Use the pumping lemma to show the following languages are context free:
 - $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$
 - $\{0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\}$
 - $\{w\#t \mid w \text{ is a substring of } t, \text{ where } w, t \in \{a, b\}^*\}$
- Give an example of a language that is not context-free, but that acts like a CFL in the pumping lemma. Prove that your example works.
- Convert the PDA $P = (\{p, q\}, \{0, 1\}, \{X, Z_0\}, \delta, q, Z_0)$ to a CFG, if δ is given by:
 - $\delta(q, 1, Z_0) = \{(q, XZ_0)\}$
 - $\delta(q, 1, X) = \{(q, XX)\}$
 - $\delta(q, 0, X) = \{(p, X)\}$
 - $\delta(q, \epsilon, X) = \{(q, \epsilon)\}$
 - $\delta(p, 1, X) = \{(p, \epsilon)\}$
 - $\delta(p, 0, Z_0) = (q, Z_0)$
- Let L be a CFL. Prove whether CFL's are closed under the following operations:
 - $max(L) = \{w \in \Sigma^* \mid w \in L, \text{ and for no } x \text{ other than } \epsilon \text{ is } wx \in L.\}$
 - $half(L) = \{w \in \Sigma^* \mid \exists x \in \Sigma^*, |x| = |w|, \text{ and } wx \in L\}$
- Modify the CYK algorithm so can report the number of distinct parse trees for the given input, rather than just reporting membership in the language.