University of Illinois at Urbana-Champaign
Department of Computer Science

Homework Assignment 1

CS 273 Introduction to Theoretical Computer Science
Fall Semester, 2002

Due: Thursday, Sept 19, at the beginnning of class

Please **read** the comments on the web page about **how to do homeworks** before doing this homework. **This is a group homework. In all the problems, you must explicitly state your reasoning to get credit.**

1. **(10 pts)**
   (a) **(5 pts) How many ways can ten boys and five girls stand in a *row* with no two girls standing beside each other?**
   (b) **(5 pts) How many ways can ten boys and five girls stand in a *circle* with no two girls standing beside each other?**

2. **(16 pts) There are five male students and five female students register for a 497 seminar course. Unfortunately, the professor has trouble hiring any graders and he himself finds the job unacceptably boring. As a result, he asks the students to exchange homeworks so that no one grades his or here own submission.**
   (a) **(4 pts) In how many ways can this be accomplished?**
   (b) **(4 pts) In how many ways can at least three students get lucky to grade their own homeworks?**
   (c) **(8 pts) Now back to the conditions given in the original problem and let the total number of registered students be n, in how many ways can this be accomplished (without the constraints in b)?**

3. **(20 pts) Every day a student walks from her home to school, which is located 10 blocks east and 14 blocks north from home. She always takes a shortest walk of 24 blocks. Assume she lives on a prefect grid.**
   (a) **(5 pts) How many different walks are possible?**
   (b) **(5 pts) Suppose that 4 blocks east and 5 blocks north of her home lives her best friend, whom she meets each day on her way to school. Now how many different ways are possible?**
   (c) **(5 pts) Suppose, in addition, that 3 blocks east and 6 blocks north of ther friend's house there is a park where the two girls stop each day to rest and play. Now how many different walks are there?**
   (d) **(5 pts) Stopping at a park to rest and play, the two students often get to school late. To avoid the temptation of the park, our two students decide never to pass the intersection where the park is. Now how many different walks are there?**

4. **(10 pts) Use *combinatorial* reasoning to prove the identity (in the form given)**

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}$$

5. (10 pts) Use *combinatorial* reasoning to prove

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$$

for any $n \geq 0$

6. (10 pts) A thousand balloons are to be given to 200 children $C_1, C_2, \ldots, C_{200}$**. In how many ways can it be done if**

(a) **(2 pts) the balloons are identical?**

(b) **(2 pts) the balloons are all different?**

(c) **(3 pts) the balloons are identical and each child must get at least one?**

(d) **(3 pts) the balloons are all different and each child must get at least one?**

7. (24 pts) Consider strings of 7 digits (e.g., 0000000, 1345692)

(a) **(4 pts) How many are there?**

(b) **(5 pts How many contain at least one 2?**

(c) **(7 pts) Suppose there are exactly one 2,two 5's and four 7's in any string. How many such strings are there?**

(d) **(8 pts) Suppose there are at most one 2's, three 5's and five 7's in any string. How many such strings are there?**