University of Illinois at Urbana-Champaign
Department of Computer Science

Homework Assignment 3

CS 273 Introduction to Theoretical Computer Science
Fall Semester, 2002

Due: Thursday, Oct. 17, at the beginnning of class

This is an individual homework. You must turn in your own work to receive credit. Please staple your homework in four pairs. 1 and 2, 3 and 4, 5 and 6, 7 and 8. Hand in each of them to the corresponding stack in class.

1. **Prove by induction that if the annihilator for a recurrence is $(E - b)^n$, for any positive integer, n, and constant, b, the solution to the recurrence is**

$$a_i = b^i \left(\sum_{j=1}^{n} c_j i^{j-1} \right), \forall i \in N$$

   in which the c_j are constants to be determined from initial conditions.

2. **Consider the simple recurrence $a_i = -a_{i-2}$ with specified initial conditions a_0 and a_1. By inspecting the recurrence, write down an simple expression for a_n in terms of a_0 and a_1 (the expression may contain "if" conditions). Using annihilators, solve the recurrence, and show that it yields an expression equivalent to your first one.**

3. **Solve the following recurrence: $a_i = 4a_{i-1} - 4a_{i-2} + i2^i + 2$ for the initial conditions $a_0 = 2$, $a_1 = 10$.**

4. **Consider the following family of recurrences:**

$$\sum_{k=0}^{n} \binom{n}{k} a_{i-k} = 0.$$

   Determine the general form of the solution to these recurrences for any integer $n \geq 1$. Note that no initial conditions are given; therefore, you may leave constants in the solution. Prove the correctness of your approach.

   [In all the following problems, for divide-and-conquer recurrences, full credit will be given for tight asymptotic bounds (including your reasoning). Also, partial credit may be given if you are only able to obtain distinct upper and lower bounds]

5. **Solve the following recurrence**

$$T(n) = 5T(n/4) + n \log\log n$$

6. **Solve the following recurrence**

$$T(n) = \log n + 2\sqrt{n} \cdot T(\lfloor \sqrt{n} \rfloor)$$

7. **Solve the following recurrence**

$$T(n) = T(\lfloor n - \log n \rfloor) + 1$$

8. **Solve the following recurrence**

$$T(n) = T(\lfloor n / \log n \rfloor) + \log n$$