University of Illinois at Urbana-Champaign
Department of Computer Science

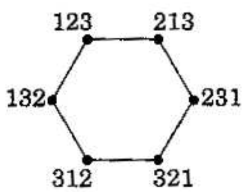
Homework Assignment 4

CS 273 Introduction to Theoretical Computer Science
Fall Semester, 2002

Due: Tuesday, Nov. 12, at the beginnning of class

This is a group homework. Please staple your homework in four pairs. 1 and 2, 3 and 4, 5 and 6, 7 and 8. Hand in each of them to the corresponding stack in class.

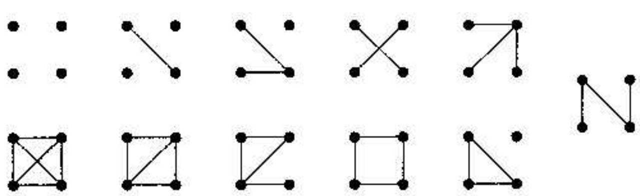
1. **(10 pts) Let G_n be the graph whose vertices are the permutations of $\{1, \ldots, n\}$ with two permutations $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$ adjacent if they differ by interchanging a pair of adjacent entries (G_3 shown below). Prove that G_n is connected.**



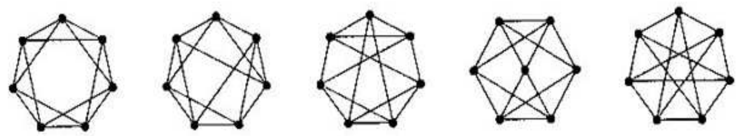
2. **(10 pts) Use induction on the number of edges to prove that absence of odd cycles is a sufficient condition for a graph to be bipartite.**

3. **(10 pts) Given that every walk of length $l-1$ contains a path from its first vertex to its last, prove that every walk of lenth l also satisfies this.**

4. **(10 pts) For a vertex set of size n, there are $2^{\binom{n}{2}}$ simple graphs. However, these graphs can be divided into disjoint isomorphism classes. For example, there are only 11 isomorphism classes for vertex set of size 4. Dertermine the number of isomorphism classes for vertex set of size 5.**



5. (10 pts) Determine which pairs of graphs below are isomorphic, presenting the proof by testing the smallest possible number of pairs.



6. (10 pts) Let G be a tree with n vertices, k leaves and maximum degree k. Prove that G is the union of k paths with a common endpoint. (The union, G, of two graphs, G_1 and G_2, is defined as $V(G) = V(G_1) \cup V(G_2)$ and $E(G) = E(G_1) \cup E(G_2)$.)

7. (10 pts) Draw and label a tree whose Prüfer sequence is

$$5, 4, 3, 5, 4, 3, 5, 4, 3$$

8. (10 pts) Draw a planar graph with 5 faces and every two faces are bordered by exactly one common edge. Or prove that such a graph does not exist.