University of Illinois at Urbana-Champaign
Department of Computer Science

Homework Assignment 6

CS 273 Introduction to Theoretical Computer Science
Fall Semester, 2002

Due: Friday, Dec. 13, at noon.

This is an individual homework. Please staple your homework in four pairs: (1), (2), (3 and 4), (5 and 6). Hand in each of them to the corresponding stack. Check the web page for the final location to hand in the assignment.

1. **For each of the following, prove whether or not the set G with the specified operation represents a group.**
   (a) **$G = \mathbb{C}$, the set of complex numbers, and the operation is standard addition of complex numbers.**
   (b) **$G = \mathbb{C}$, the set of complex numbers, and the operation is standard multiplication of complex numbers.**
2. **Follow the same instructions as for the previous problem.**
   (a) **G is the set of all binary strings of length 5, and the operation is bitwise exclusive or (XOR).**
   (b) **G is the set of all 2×2 matrices of the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $a, b, c, d \in \mathbb{R}$ and $ad - bc \neq 0$, and the operation is matrix multiplication.**
3. **Prove that if every element of a group, G, is equal to its own inverse, then G is an Abelian (commutative) group.**
4. **Let $G = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$, in which $\mathbb{Q}$ is the set of all rational numbers. Prove that G is a subgroup of $\mathbb{R}$ under the operation of addition.**
5. **Consider the group $\mathbb{Z}_{12}$ (mod 12 addition on the integers).**
   (a) **Give the inverse of every element.**
   (b) **Find all of the generators.**
   (c) **Determine the order of each element of the group.**
6. **Consider the following permutation**

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 3 & 2 & 1 & 4 & 5 \end{pmatrix}.$$

   (a) **Find all of its orbits.**
   (b) **Express the permutation as a product of 2-cycles.**