CS 397 Homework # 1
Due: Jan. 31, in class

1. (Game against nature) Let $U = \{1, 2, 3\}$ and $\Theta = \{A, B, C, D\}$ for a game against nature. The matrix,

$$\begin{pmatrix} -1 & -1 & 1 \\ 3 & 0 & 5 \\ 2 & 7 & 5 \\ -1 & -1 & -2 \end{pmatrix},$$

gives the loss for each combination of choices by the decision maker and nature. Suppose the nature chooses a row, and the decision maker chooses a column. Suppose that for nature it is known that $P(A) = \frac{1}{5}$, $P(B) = \frac{2}{5}$, $P(C) = \frac{1}{10}$, $P(D) = \frac{3}{10}$. a) Use nondeterministic reasoning to find the minimax decision and worst-case loss. b) Use probabilistic reasoning to find the best expected-case decision and expected loss.

2. (Hurwitz decision) Many other reasonable decision rules are possible. The answer to Problem 1.a reflects extreme pessimism. a) Suppose that extreme optimism is used. Select the choice that optimizes the best-case loss. b) One approach is to develop a coefficient of optimism, $\alpha \in [0, 1]$, which allows one to interpolate between the two extreme scenarios. Thus, a decision, $u \in U$ is chosen by minimizing

$$\alpha \max_{\theta \in \Theta} L(u, \theta) + (1 - \alpha) \min_{\theta \in \Theta} L(u, \theta).$$

Give the optimal decision for this scenario under all possible choices for $\alpha \in [0, 1]$. Give your answer as a list of choices, each with a specified range of α.

3. (Minimum Doh![1]) Suppose that after making a decision, you observe the choice made by nature. How does the loss that you received compare with the best loss that could have been obtained if you chose something else, given this choice by nature? This difference in losses can be considered as *regret*. Psychologists have argued that some people make choices based on minimizing regret. It reflects how badly you wish you had done something else after making the decision. a) Develop an expression for the worst-case regret, and use it make a minimax regret decision using the matrix from Problem 1. b) Develop an expression for the expected regret, and use it to make a minimum expected regret decision.

4. (Reverse justification of priors) For the matrix given in Problem 1, consider the set of all probability distributions for nature. Characterize the set of all distributions for which the minimax decision and the best expected decision will result in the same choice.

5. (Bayesian decision theory) Consider a Bayesian decision theory scenario with loss function L. Show that the decision rule never changes if we replace $L(u, \theta)$ by $aL(u, \theta) + b$, for any $a > 0$ and $b \in \mathbb{R}$.

6. (Bayesian classification) Suppose that there are two classes, $\Omega = \{\omega_1, \omega_2\}$, with $P(\omega_1) = P(\omega_2) = \frac{1}{2}$. The observation space, Y, is $\mathbb{R}$. Recall from probability theory that the normal (or Gaussian) probability density function is

$$p(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/2\sigma^2},$$

in which μ denotes the mean, and σ^2 denotes the variance. Suppose that $p(y|\omega_1)$ is a normal density in which $\mu = 0$ and $\sigma^2 = 1$. Suppose that $p(y|\omega_2)$ is a normal density in which $\mu = 6$ and $\sigma^2 = 4$. Find the optimal classification rule, $\gamma : Y \to \Omega$. You are welcome to solve the problem numerically (by computer) or graphically (by careful function plotting). Carefully explain how you arrived at the answer in any case.

[1] The Homer Simpson term "Doh!" was added last year to the Oxford English Dictionary as an expression of regret.