IROS 2009 Tutorial: Filtering and Planning in I-Spaces PART 5: PLANNING

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Overview of Topics

From filters to planning
General issues
Maze searching
Gap navigation trees
Learning convex hulls of landmarks
Bug algorithms
Sensorless manipulation

1. An I-space view of planning, starting with temporal filters
2. General planning issues
3. Maze searching
4. Gap navigation trees
5. Landmark-based navigation
6. Bug algorithms
7. Sensorless manipulation

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From filters to planning

Using Filters For Planning

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Let $\mathcal{I}$ be any I-space.

Assume a filter

$$\iota_k = \phi(\iota_{k-1}, u_{k-1}, y_k)$$

is given.

Let $G \subset \mathcal{I}$ be a *goal region*.

Starting from ι_0, what sequence of actions u_1, u_2, . . ., will lead to some future I-state $\iota_k \in G$?

Using Filters For Planning

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The future may be unpredictable.

Introduce an *I-state dependent* plan:

$$\pi : \mathcal{I} \rightarrow U$$

Using a filter ϕ, the execution of a plan can be expressed as

$$\iota_k = \phi(\iota_{k-1}, y_k, \pi(\iota_{k-1}))$$

The I-space $\mathcal{I}$ is just a sort of “C-space” that is being explored.

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General issues

Main Issues

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The following issues arise repreatedly in planning:

1. **Predictability**
2. **Reachability**
3. **Optimality**
4. **Computability**

Predictability

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Are the effects of actions predictable in the I-space $\mathcal{I}$?

If **yes**, then a *path* through the I-space is obtained.

Example: Sensorless manipulation
Example: Visibility-based pursuit evasion

By analogy to path planning in C-space:

1. Combinatorial planning in I-space
2. Sampling-based planing in I-space

If **no**, then information feedback is critical
It is like feedback planning (or control) in C-space, but instead over I-space

Reachability

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Reachability:
Is the goal region $G \subset \mathcal{I}$ even reachable from the initial I-state?

Do there even exist actions that will take us to G?

Does there exist a plan that can reach G?

With unpredictability, is G *guaranteed* to be reached, over all possible disturbances?

A more basic question is whether the goal can even be adequately expressed in $\mathcal{I}$.

Optimality

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Perhaps many plans can reach G

What criteria should be formulated to compare plans?

Which plans are the best, or optimal with respect to criteria?

Do optimal plans even exist?

Computability

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Given a description of the problem, can an algorithm be determined that automatically computes a useful plan?

Sometimes a clever human designs the plan (e.g. bug algorithms)

What is the algorithmic complexity of computing a solution plan?

What is the implementation difficulty of computing a solution plan?

Important Generic Examples

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State feedback: I-space is $\mathcal{I} = X$ and plan is $\pi : X \rightarrow U$

Open loop: $\mathcal{I} = \mathbb{N}$ and $\pi : \mathbb{N} \rightarrow U$
π can be written as $(u_1, u_2, u_3, \ldots)$

Sensor feedback: $\mathcal{I} = Y$ and $\pi : Y \rightarrow U$

History feedback $\mathcal{I} = \mathcal{I}_{hist}$ and $\pi : \mathcal{I}_{hist} \rightarrow U$

Recall the previous filters over these 4 I-spaces.

Now we move from *passive* to *active*.

Overall Process

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Based on the task, an overall approach that leads to planning:

1. Design the system, which includes the environment, bodies, and sensors.

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2. Define the models, which provide the state space X, the sensor mapping h, and the state transition function f.

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3. Select an I-space $\mathcal{I}$ for which a filter ϕ can be practically computed.

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4. Take the desired goal, expressed over X, and convert it into an expression over $\mathcal{I}$.

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3. Select an I-space $\mathcal{I}$ for which a filter ϕ can be practically computed.
4. Take the desired goal, expressed over X, and convert it into an expression over $\mathcal{I}$.
5. Compute a plan π over $\mathcal{I}$ that achieves the goal in terms of $\mathcal{I}$.

Really, all steps should be considered together.

Might have to backtrack.

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Maze searching

Maze Searching

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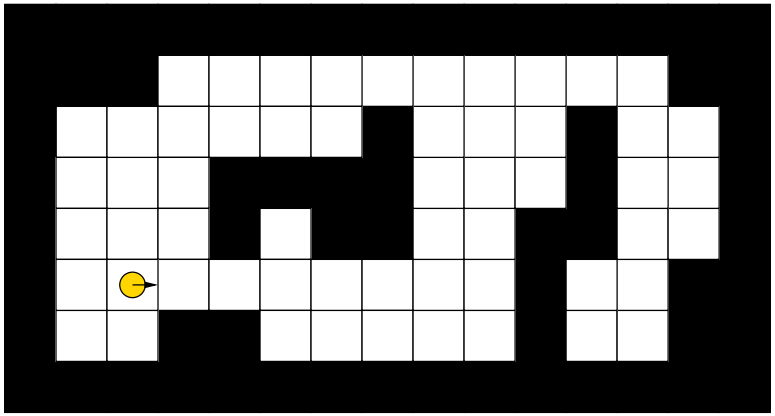

Each $E \in \mathcal{E}$ is a bounded set of white tiles.

$$X \subset \mathbb{Z} \times \mathbb{Z} \times D \times \mathcal{E}$$

Actions: 1) Rotate 90 degrees CCW; 2) Move foward one tile.

Task: Make a plan that systematically searches all white tiles.
For example, find a hidden treasure.

Maze Searching: Simple Mapping

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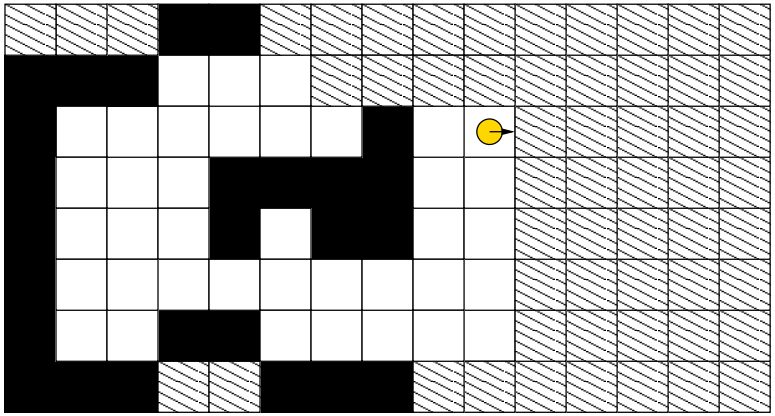
Could try $\mathcal{I} = \mathrm{pow}(\mathbb{Z} \times \mathbb{Z} \times D \times E)$.
Too large!

Instead, maintain I-states B (known black tiles) and W (known white tiles).



All other tiles assumed “unknown”.

I-space $\mathcal{I}$ is all ways to partition $\mathbb{Z} \times \mathbb{Z}$ into connected “white”, “black”, and “unknown” tiles.
Linear space required for an I-state (filter memory).

Maze Searching: Blum and Kozen 1978

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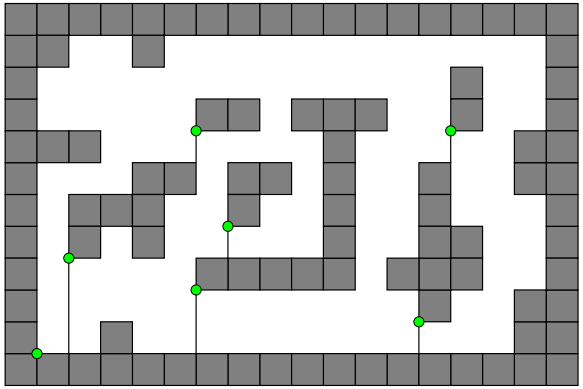

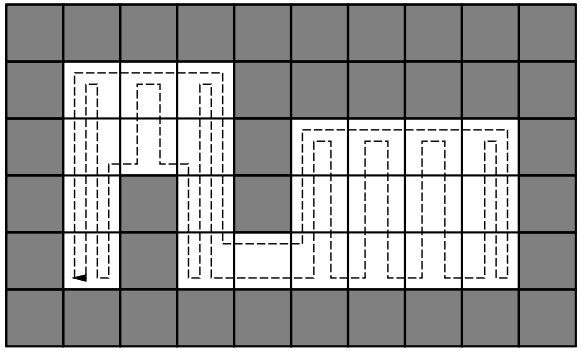

I-state: Latitude (integer) and orientation (two bits)

Only logarithmic space required: Not enough for a “map”.

They found an I-space that is much smaller than the set of all maps.

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Gap navigation trees

Make Gap Navigation Trees Active

From filters to planning

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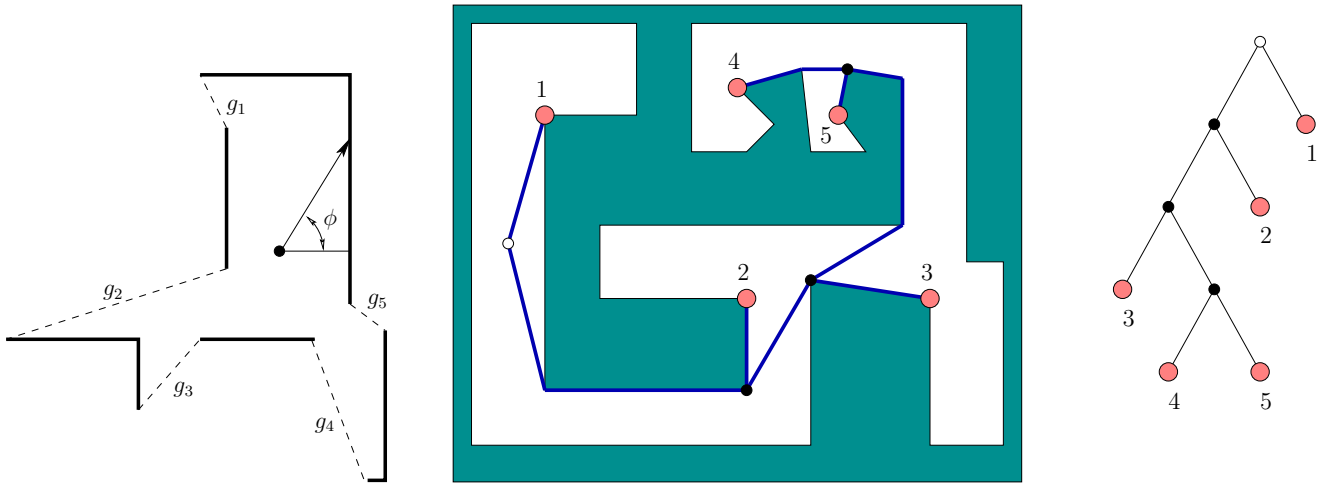

For gap navigation trees, two active tasks:

1. Full exploration of the environment
2. Distance-optimal navigation to retrieve objects

Tovar, Murrieta, LaValle, *IEEE Trans. Robotics*, 2007.

Introduce an Action

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A gap-chasing action is introduced:

Move the robot toward a gap g until a critical event occurs.

One of two events must occur:

1. The gap g splits into two gaps g' and g''.
2. The gap g disappears.

Full Exploration

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If a gap ever appears, mark it as *primitive*.

This is an extension to the filter I-state.

1. Mark all gaps in the initial tree as *non-primitive*.
2. Let $k = 1$.
3. Chase any gap g that is a non-primtive leaf.
4. If g *disappears*, then go to Step 6.
5. If g *splits*, then chase one of its children.
6. Unless all leaves are primitive, increment k and go to Step 3.

At the end, all leaves are primitive and the environment has been fully explored.

Simple Illustration

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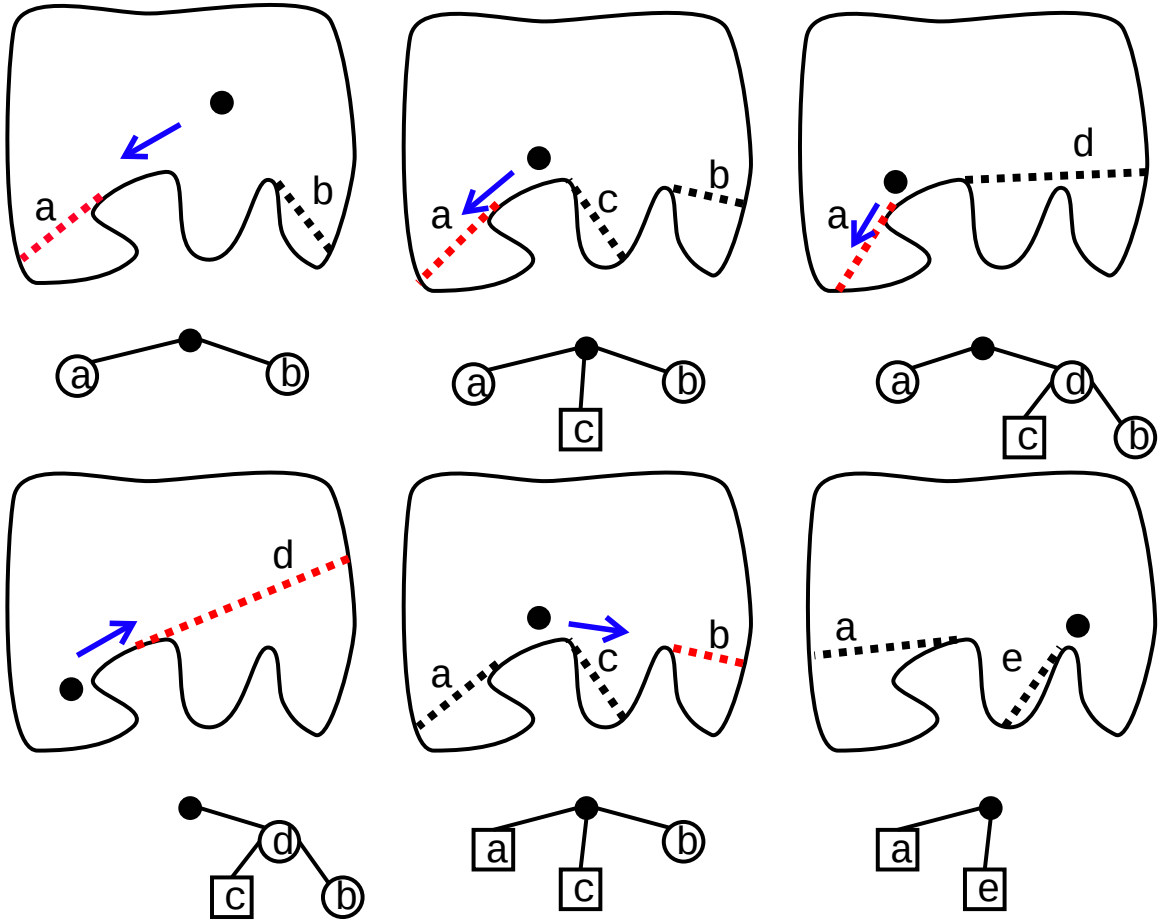
Chase every non-primitive leaf:



Eventually, all leaves become primitive.

Optimal Object Retrieval

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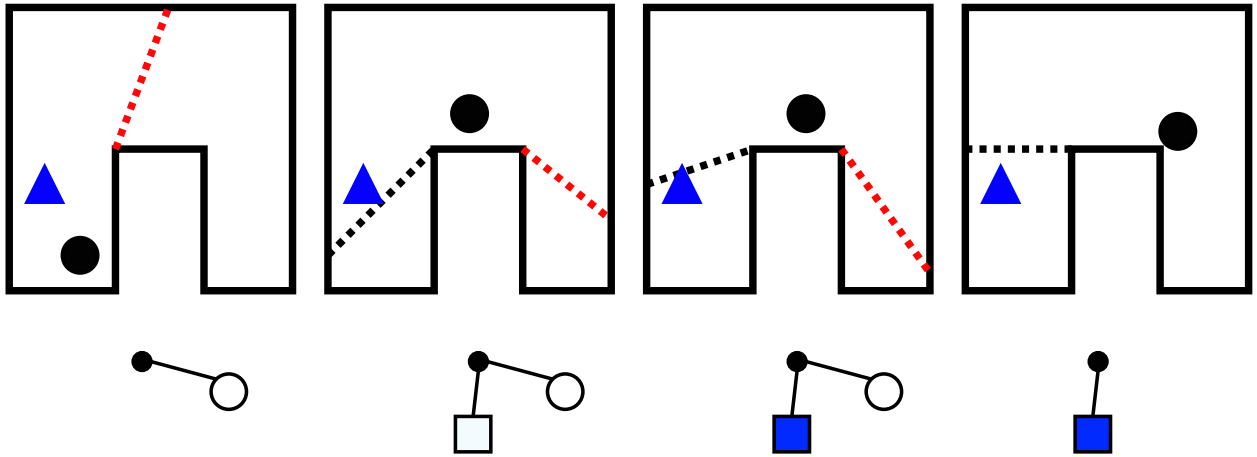
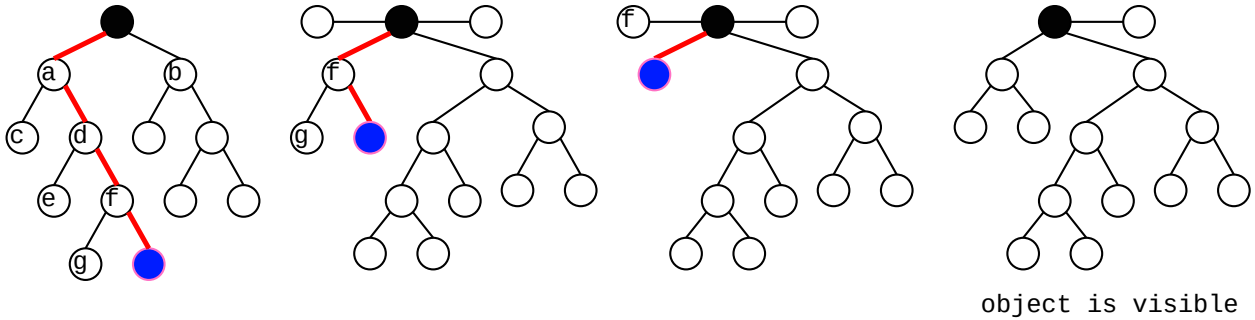
There are no coordinates.



Objects hide behind gaps.



Chase the appropriate sequence of gaps.

Possible Current States

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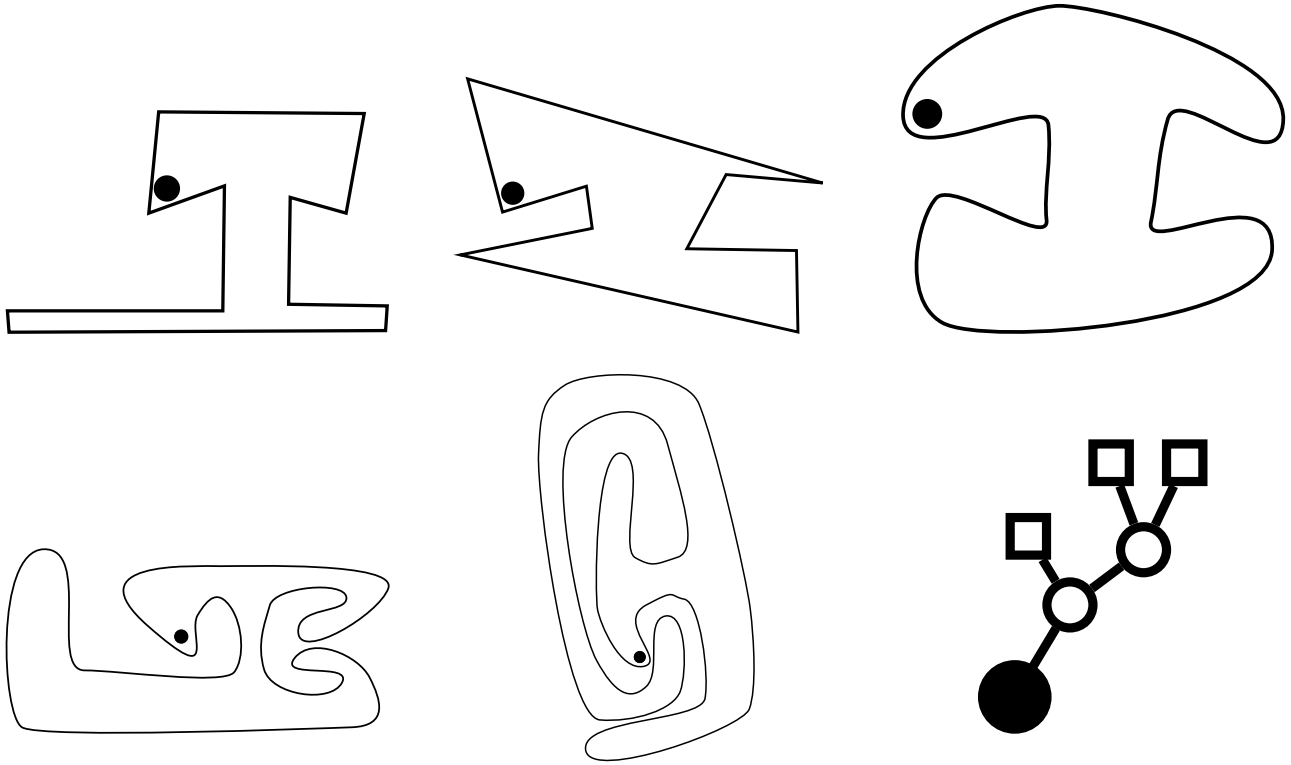

Many configuraton-environment pairs have the same tree.

The robot does not have to distinguish!

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Learning convex hulls of landmarks

Recall: Cyclic Permutation Sensor

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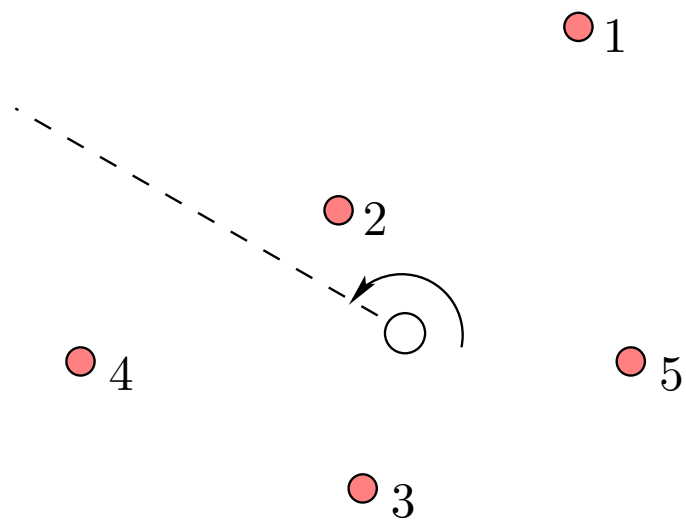
Relation: “is to the left of in counterclockwise order”



Observation: $y = (1, 2, 4, 3, 5)$

Making an Active Version

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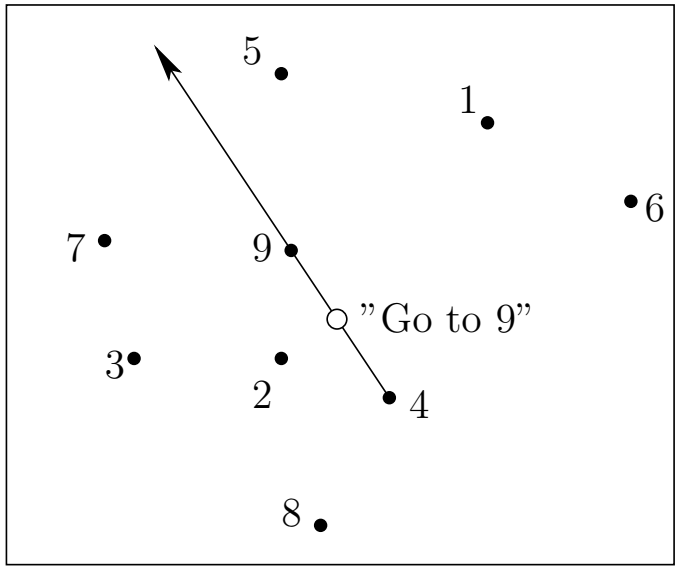
Tovar, Freda, LaValle, 2007.

- Landmark locations are unknown
- Introduce action: "Go to landmark i"
- Can notice which landmarks are "to the left" of the path.



Sense that $(6, 1, 5)$ is to the right of $(7, 2, 3, 8)$.

Learning the Arrangement

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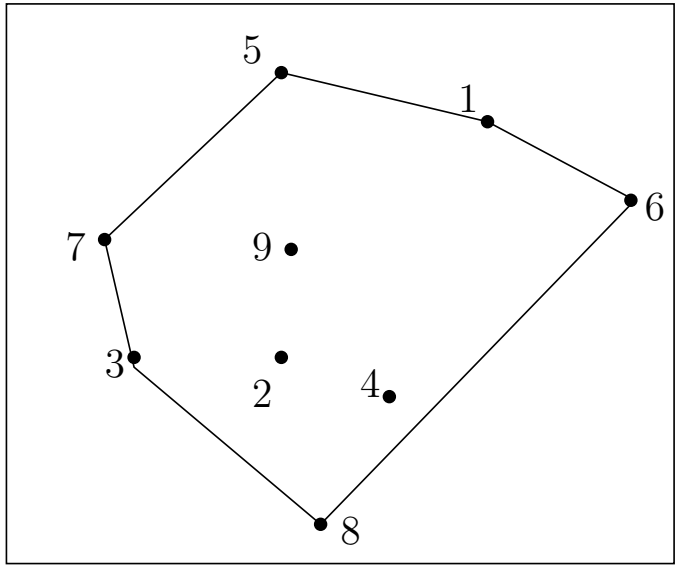

By visiting all pairs, the filter can learn:

- For any subset $L' \subset L$ of landmarks, which others in L lie in the convex hull of L'.
- Equivalently, the robot learns the dual arrangement, order types, oriented matroid.
- The robot can navigation to any goal specified as a cyclic permutation.

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Bug algorithms

Recall Bug Algorithms

From filters to planning

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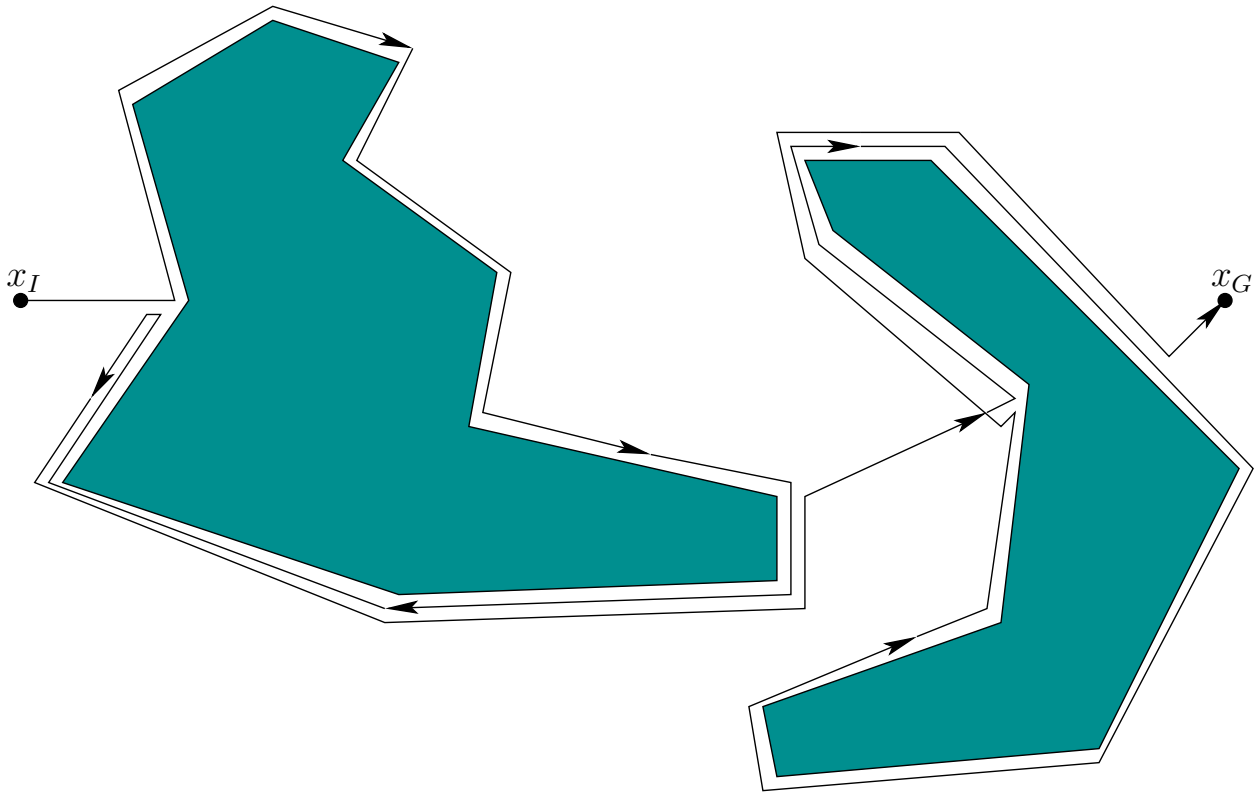

Navigate without being given an initial map E

Lumelsky, Stepanov, 1987; Kamon, Rivlin, Rimon, 1997; many others...

Intensity Bug

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Taylor, LaValle, ICRA 2009

- The plane contains unknown obstacles with piecewise analytic boundary.
- Each obstacle boundary has finite length.
- A *tower* sends a constant signal.
- Robot has very limited sensors.
- Command the robot so that it reaches the tower.

Sensors

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- **Boundary (or Contact) sensor:**
  Indicate whether or not robot is on the boundary.
- **Tower alignment/gradient sensor:**
  Indicate whether robot is facing the tower (or intensity gradient).
- **Transformed signal intensity sensor:**
  Observe the value of $m(p - p_t)$.

Regarding m:

- m has only only local maximum, at the tower.
- The function m itself is not given.
- Level sets of m may be symmetric (circles) or asymmetric.

Actions

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There are three possible actions:

u_{fwd}: Go straight until either ∂E is hit, tower is hit, or local intensity maximum detected.

u_{fol}: Follow ∂E until local maximum detected.

u_{ori}: Rotate until facing tower (or local gradient).

An Plan Designed by Humans

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Illustration of the Plan

Guaranteed to converge; upper bound on distance shown.

Information Spaces

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State space: $X \subset \mathbb{R}^2 \times S^1 \times \mathcal{E}$

I-space: $\mathcal{I} = Y^3 \subset \mathbb{R}^3$

I-state components:

1. Current observation
2. Observation when obstacle was last contacted
3. Observation just prior to application of u_{fwd}

Multiple Iterations in the Interior

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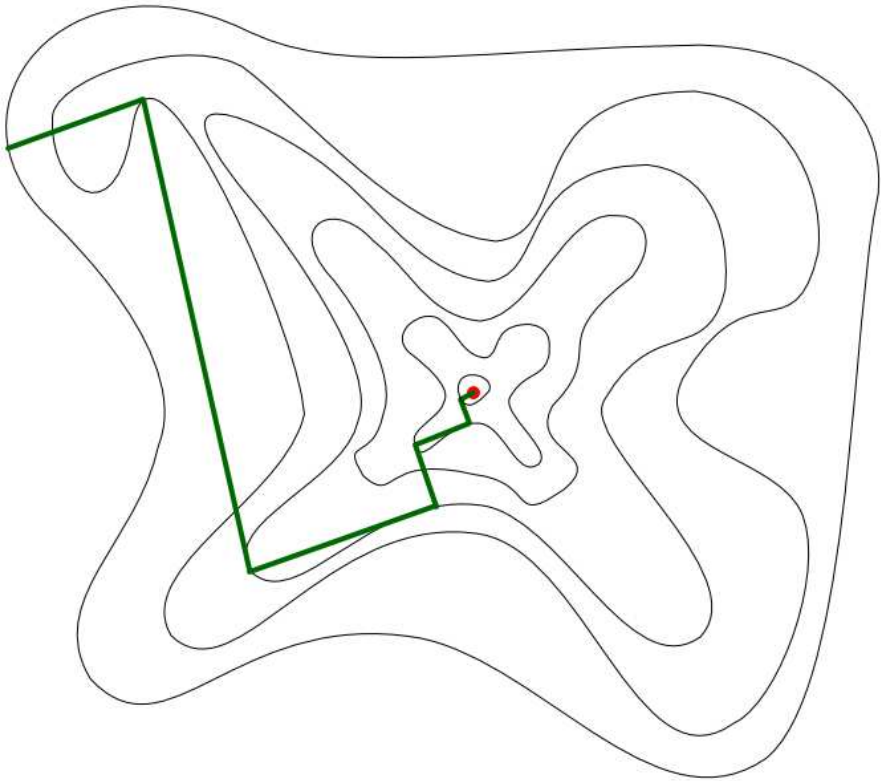

- Equivalent to Steepest Descent with Line Search.
- Result: Convergence is obtained, but distance bound depends on properties of m.

Decidability

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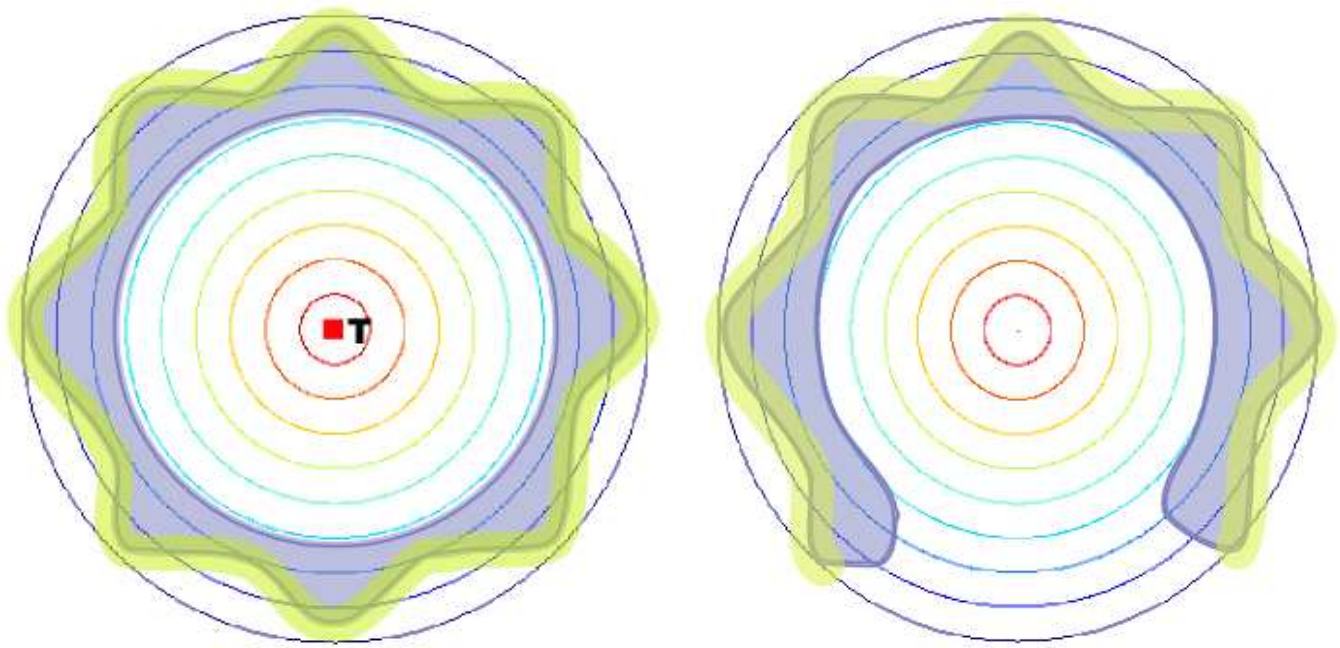
Proposition: Using its sensors and motion primitives, it is impossible for the robot to determine whether the tower is reachable, in other words whether $p_t \in E$.



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Sensorless manipulation

Sensorless Manipulation

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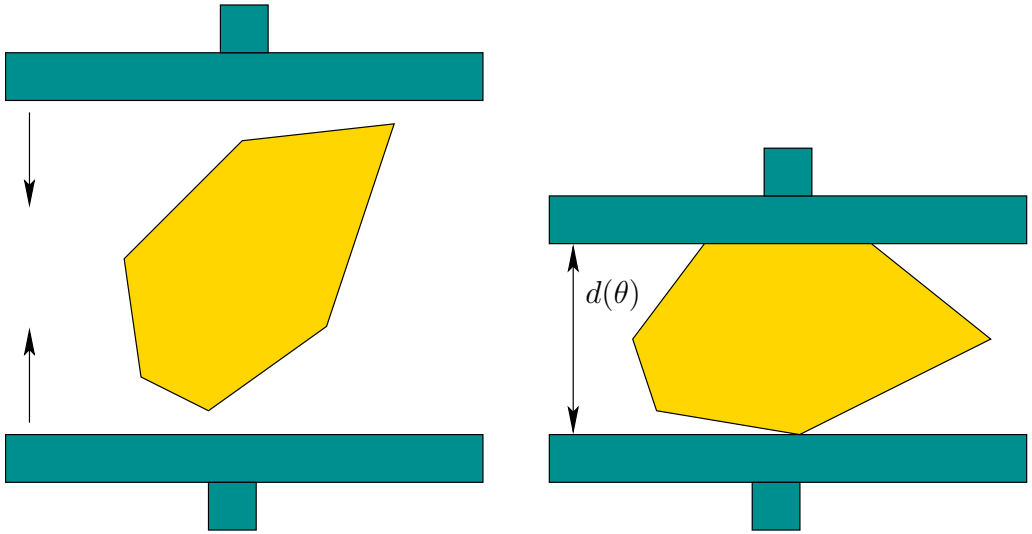
Try to force parts into a known orientation



Mason, Goldberg, 1990

$$X = S^1 \quad \mathcal{I} = \mathrm{pow}(X)$$

Plan: $\pi = (u_1, u_2, \ldots, u_n)$
A sequence of squeeze operations

Sensorless Manipulation

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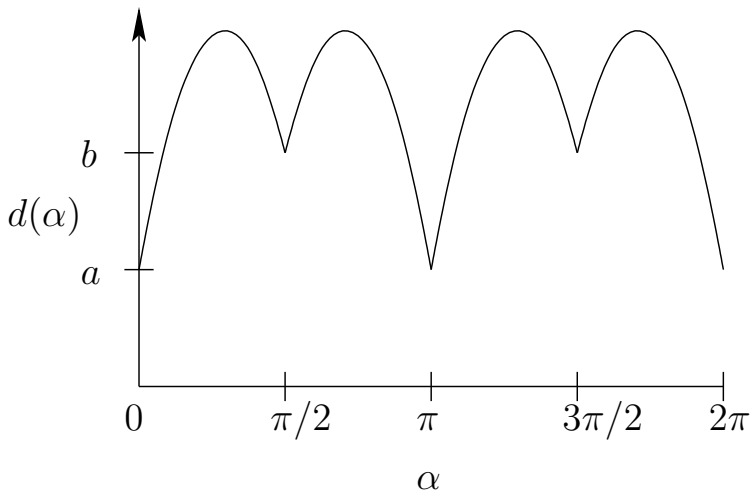

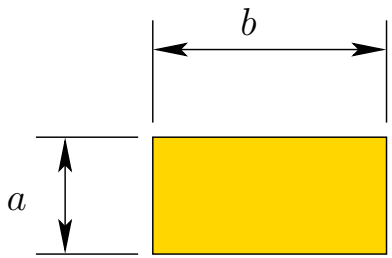

Consider the “diameter” as a function of orientation.

Sensorless Manipulation

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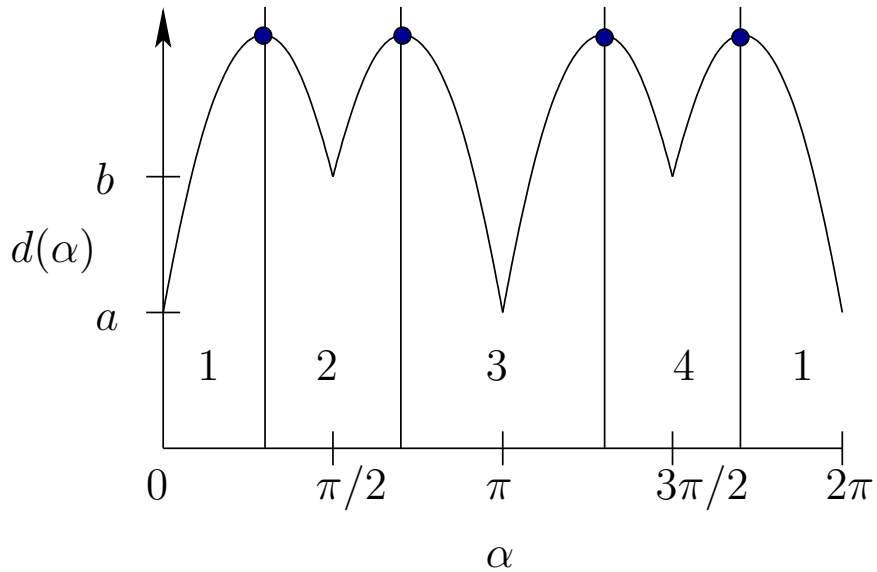

There are four regions of attraction.

This causes a funneling effect when actions are applied.

Sensorless Manipulation

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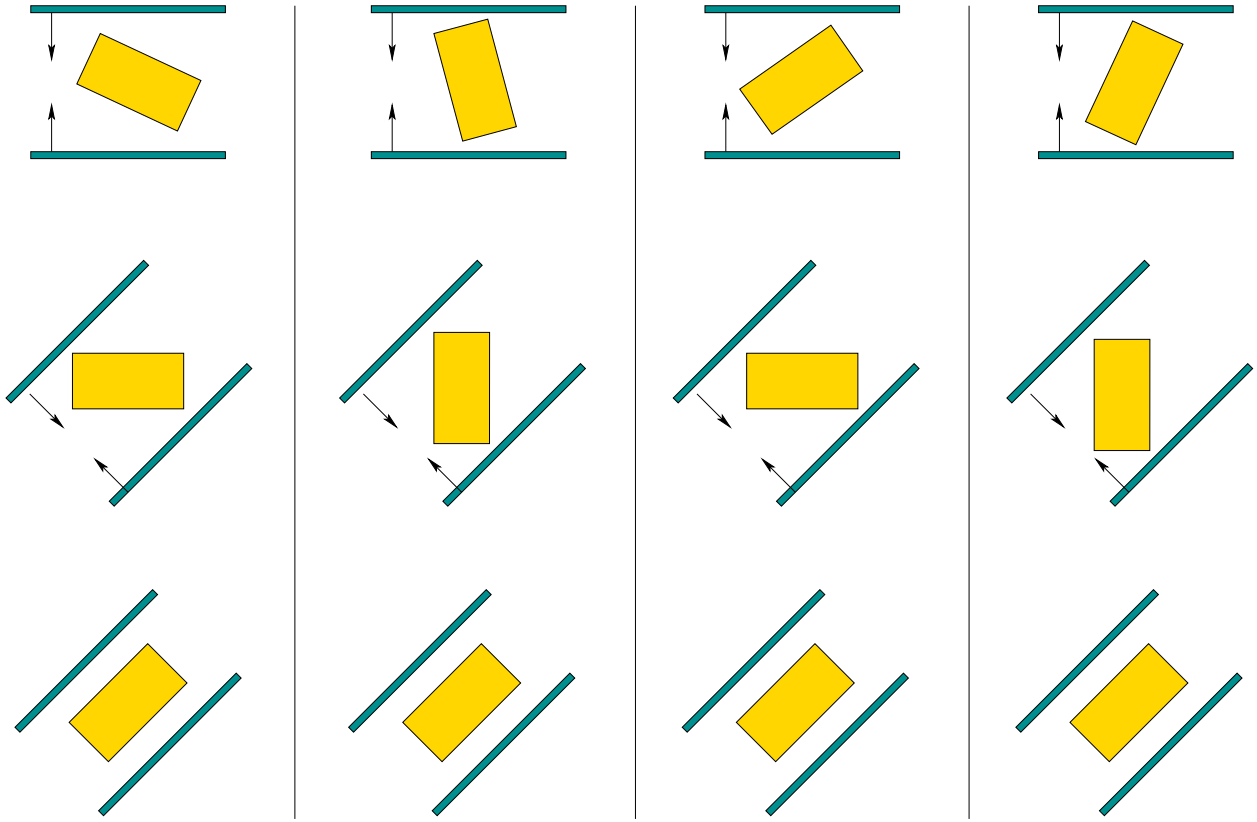

A computed plan that applies two squeeze actions

Part 5 Summary

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- Use filters to make I-space transitions
- Plan directly in the I-space
- General planning issues
- Need to design virtual sensors, filters, and planning around a task
- Several examples were shown

Although several examples of nice reduced-complexity I-spaces have been found, we have barely scratched the surface...