Umeå

\[ S^2 \times S^1 \times S^1 \times S^1 \rightarrow \text{car has } 5 \text{ degrees of freedom} \]

\[ R^3 \times S^1 \rightarrow 4 \text{ d.o.f.} \]

\[ \text{SO}(3) \rightarrow \text{RP}^3 \rightarrow 3 \text{ degrees of freedom} \]
Topological Concepts (LaValle, 4.1)

Topological space

A set $X$ is called a topological space if there is a collection of subsets of $X$, called open sets, such that:

1) The union of a countable number of open sets is an open set.
2) The intersection of a finite number of open sets is an open set.
3) $X$, $\emptyset$ are open sets.
A subset $C \subseteq X$ is called **closed** if $X - C$ is an open set.

$\mathbb{R}$

- $(0, 1)$ - open set
- $(0, 1) \cup (3, 4)$

- $[0, 1]$ - closed set
- $\mathbb{R}$ - closed set
- $\phi$ - closed, open
- $X$ - open, closed

$[0, 1]$ - neither open nor closed
A function is **continuous** if \( f^{-1}(O) \) is an open set for any open set \( O \).

\[ f: X \rightarrow Y \quad f^{-1}(O) = \{ x \in X \mid f(x) \in O \} \]

preimage
The composition of two continuous functions is a continuous function.
A function $f: X \rightarrow Y$ is called a **homeomorphism** if $f$ is bijective, $f$ is continuous, $f^{-1}$ is continuous.
Manifold $M$

A top. space is a manifold if for any $x \in M$, there exists an open set $O \subset M$ such that:

1) $x \in O$, 2) $O$ is homeomorphic to $\mathbb{R}^n$, 3) $n$ is fixed for all $x$

$n$ is called the dimension of $M$