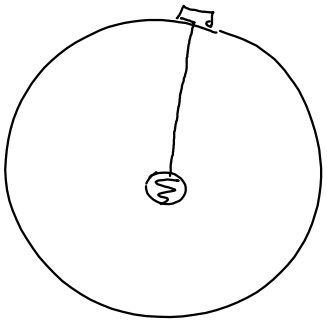
Umeå

$S^2 \times \cancel{S^1} \times S^{-1} \times S^1 \rightarrow$ car has 5 degrees of freedom

$\mathbb{R}^3 \times \cancel{S^1} \longrightarrow$ 4 d.o.f.

$SO(3) \rightarrow RP^3 \longrightarrow$ 3 degrees of freedom

$S^2 \times \cancel{S^1}$



Topological Concepts (LaValle, 4.1)

Topological space

A set X is called a topological space if there is a collection of subsets of X, called open sets, such that:

1) The union of a countable number of open sets is an open set.
2) The intersection of a finite number of open sets is an open set
3) X, ϕ are open sets

A subset $C \subset X$ is called <u>closed</u> if $X - C$ is an open set

$\mathbb{R}$

$(0,1)$ - open set

$(0,1) \cup (3,4)$

$[0,1]$ - closed set

$\{1\}$ - closed set

$(0,1]$ - neither open nor closed

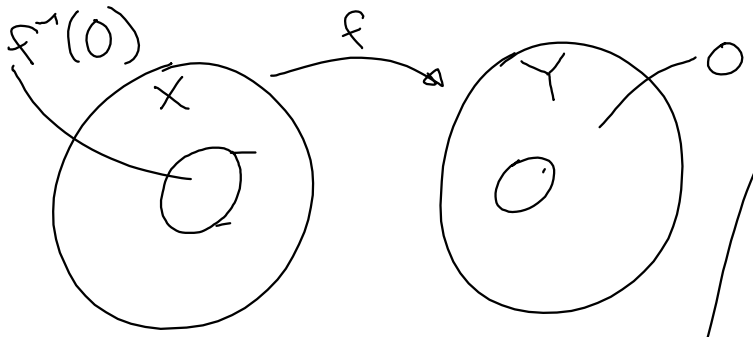
X - open, closed

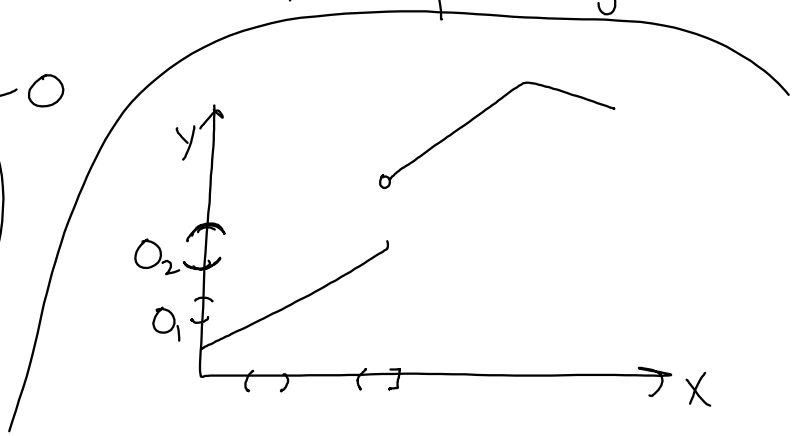
ϕ - closed, open

A function is <u>continuous</u> if $f^{-1}(O)$ is an open set for any open set O

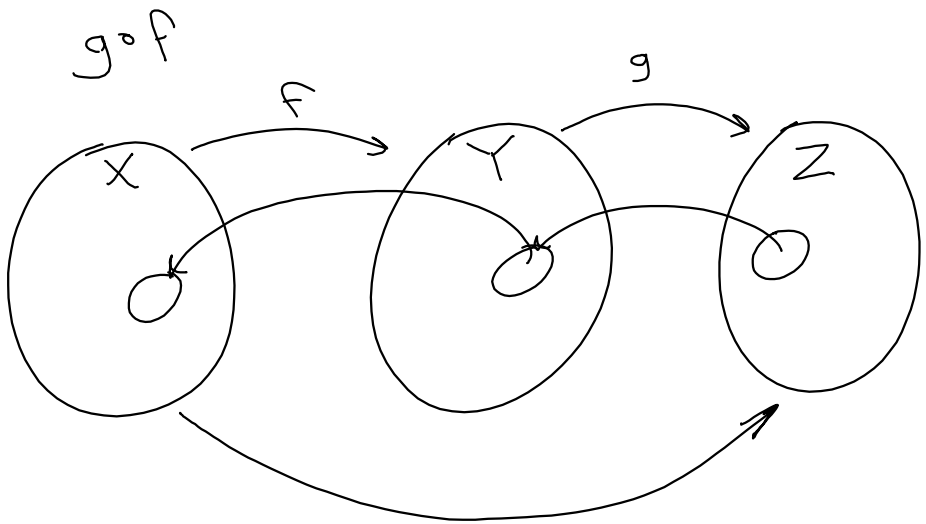
$\parallel$

$f: X \to Y$ $\qquad f^{-1}(O) = \{x \in X \mid f(x) \in O\}$ preimage



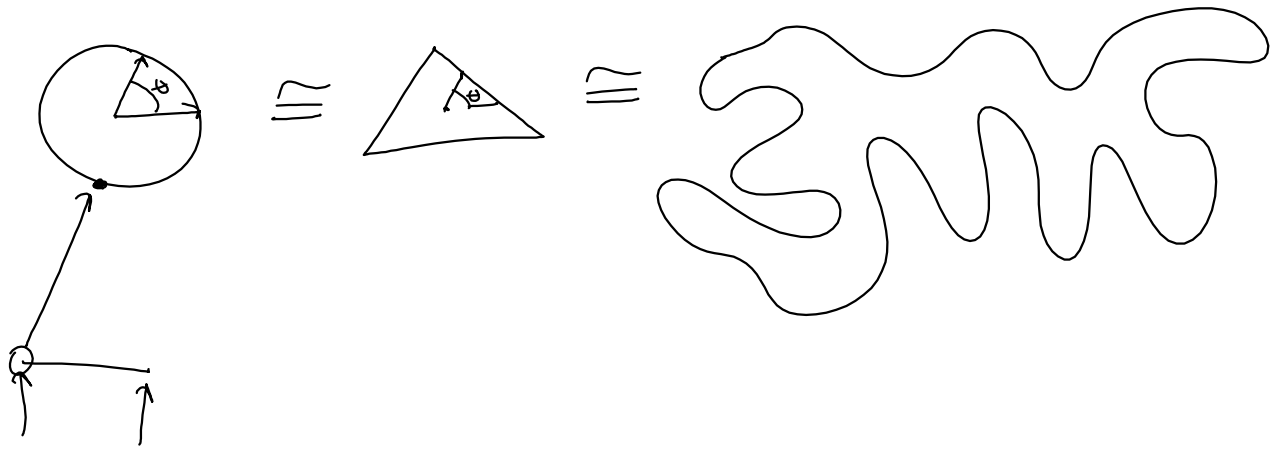


The composition of 2 ~~f~~ continuous functions is a continuous function

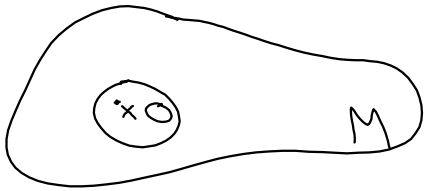


A function $f: X \to Y$ is called a <u>homeomorphism</u> if f is bijective, f is continuous, f^{-1} is continuous

<u>Ex</u>

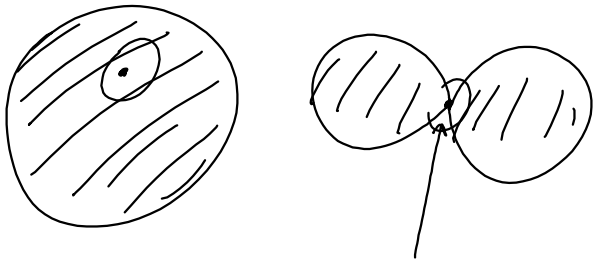


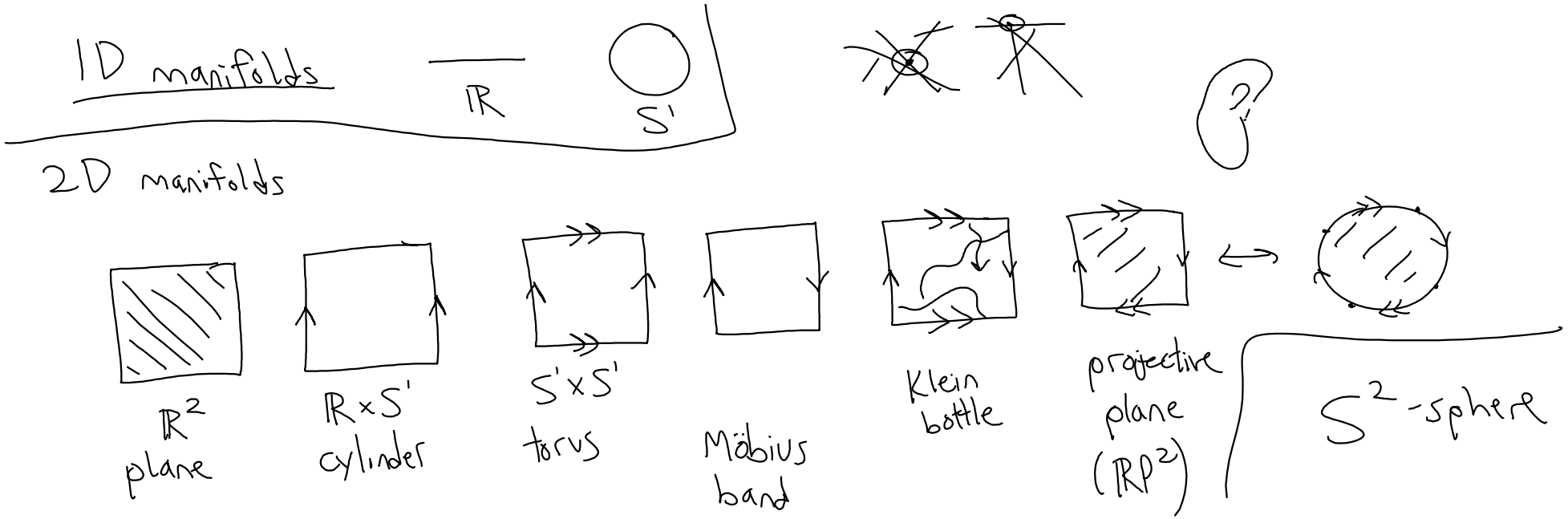
<u>Manifold</u>



A top. space M is a <u>manifold</u> if for any $x \in M$, there exists an open set $O \subset M$ such that: 1) $x \in O$, 2) O is homeomorphic to $\mathbb{R}^n$, 3) n is fixed for all x.

n is called the <u>dimension</u> of M




1D manifolds
ℝ
S¹
?
2D manifolds
ℝ²
plane
ℝ×S¹
cylinder
S¹×S¹
torus
Möbius
band
Klein
bottle
projective
plane
(ℝP²)
S²-sphere