Interesting example: American professor in Rovaniemi

Goal: cross the Arctic circle

Success: compass, clock

Actuator: bicycle/legs

Cost functional

\[ \pi: I \rightarrow \mathcal{U} \]

state estimation \[ \pi: \text{state} \rightarrow \mathcal{U} \]

time estimation \[ \pi: \text{time} \rightarrow \mathcal{U} \]

select a plan that optimizes the cost

\[ \pi: \text{Pow}(X) \rightarrow \mathcal{U} \]
Sensing and Actuation Problems on Grids

We control a mobile robot in $\mathbb{R}^2$, but discretized.

Define state space $X$.

Let the set $X \times X$, which contains elements of the form $(i,j)$, be considered as a collection of tiles.

Let $\mathcal{E} \subseteq X \times X$ be a finite, connected subset.

A tile in $\mathcal{E}$ is called white; otherwise, it is called black.

Let $D = \{\uparrow, \downarrow, \leftarrow, \rightarrow\}$. 
Action space \( U = \{F, B, R, L\} \)

- **F**: move forward one tile, maintain orientation
- **R**: turn right (90°), move forward one tile
- **L**: turn left, move forward one tile
- **B**: turn 180°, move forward 1 tile

If the robot is blocked by a black tile, then the position is unchanged.

Define a sensor

\[
X = e \times D
\]

Rather than \( h : X \times X \rightarrow Y \), we have a history-based sensor of the form \( h : X \times X \rightarrow Y \)

Let \( Y = \{0, 1\} \)

- \( 1 \): robot changed position
- \( 0 \): otherwise

\[
h(x_{k-1}, x_k) = \begin{cases} 1 & \text{if } p_{k-1} \neq p_k \\ 0 & \text{otherwise} \end{cases}
\]
Task: Localize the robot
Move until the state is known, or the set of possible
states is as small as possible.

\[ \text{Pow}(X) \rightarrow \text{sets of possible positions and directions} \]

In \( \text{Pow}(X) \) we want to reach a singleton
Imagine keeping track of \( X(\tau_k) \) as the robot moves
\[ X_0 = X \quad X(\tau_1) \rightarrow X(\tau_2) \rightarrow X(\tau_3) \rightarrow \ldots \]

Interesting property:
\[ |X(\tau_k)| \leq |X(\tau_{k-1})| \quad \text{under all conditions} \]
This means that a simple systematic search should work
\[ \text{avoid pointless cycles} \]
\[ \text{Sec. 2.2} \]
Initially, \( x^1 \) and \( y = E \).

The initial state is

\[
\tilde{x} = (x_1, x_2, x_3, x_4, x_5) = (1, 1, 1, 1, 1)
\]

Try \( \psi = (F, L, F, L) \),

\[
\psi' = (F, L, F, L)
\]
Projects

Implementation
Experimental
Software

Theoretical Formulation
Proofs Analysis
Derived Spaces

98% students
- Working in pairs is OK
- Working alone is OK

1) too easy?
2) too hard?
3) irrelevant

By tomorrow:
- Who is working with whom

By Friday, noon: Project proposal
(Few sentences)

By Friday, 3pm: responses

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Suggestions

1) Grid model, but with grey-scale values
2) Probabilistic version of grids
3) Study/implement inference problem for agents on a graph
4) Localization/Mapping problems
   - Without sensing (O’Kane, LaValle  Almost Sensorless Localization)

5) Pursuit-evasion
   - in a graph
   - in a polygon
   - grid?

6) Landmark-based problems

Look at Ch 12 papers on information spaces