Sensors with disturbances

Ψ · nature state space

$\Omega = X \times \Psi$

$h : \Omega \to Y$

ψ – we don't have direct access.

Cannot predict $\psi \in \Psi$

However, suppose y is observed and x is known

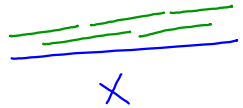
Solve $y = h(x, \psi)$ for ψ.

Preimages are larger

$H(y)$ for all $y \in Y$

forms a cover

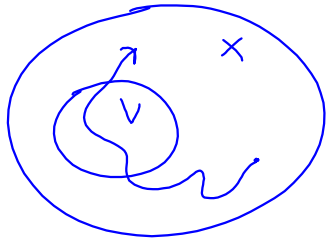
$$\bigcup_{y \in Y} H(y) = X$$



Probabilistic case

We are give $p(\psi)$ or $p(\psi|x)$

$p(\psi)$ or $p(\psi|x)$



$y = \psi \rightarrow p(y|x)$

$H(0) = X$
$H(1) = X$] nondet.

Ex A faulty (noisy) detection sensor

$V \subset X$ is detection region

$Y = \{0,1\}$ $\Psi = \{0,1\}$

Given

$p(\psi=1|x\in V)$
$p(\psi=0|x\notin V)$
$p(\psi=1|x\notin V)$
$p(\psi=0|x\in V)$

$\psi = y$

$y = h(x, \psi)$ "controlled experiment"

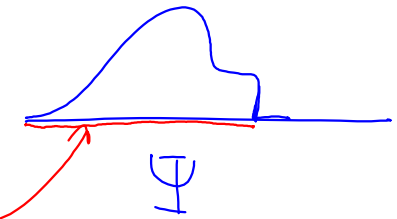
Three cases

1) simple sensor $y = h(x)$ → $\boxed{S(x)} \subseteq Y$

2) nondet. → $y = h(x, \psi)$

bounded uncertainty

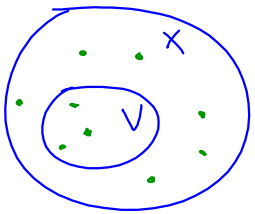
3) probabilistic → $P(y|x)$



Suppose $P(\psi|x)$ is known.

$P(y|x) = P(\psi|x)$ for the unique ψ such that $y = h(x, \psi)$

Ex Agent counter $X = \mathbb{R}^{2n}$ $Y = \{0, 1, \ldots, n\}$



Ideally, $h(x) = |A(x) \cap V|$

$A(x) \subset \mathbb{R}^2$

$\Psi = \{0, 1, 2\}$

$y = h(x, \psi) = |A(x) \cap V| + \psi$

$$P(y|x) = \begin{cases} P(\eta|x) & \text{for the unique } \eta \text{ such that } \underline{y = h(x,\eta)} \\ 0 & \text{if no such solution } (\eta) \text{ exists} \end{cases}$$

10 5

Measure temperature

$X = \mathbb{Z}$ $\quad \mathcal{Y} = \{0,1,2\}$

$Y = \mathbb{Z}$ $\quad P(10|2)$

$\underline{y = x + \eta}$ $\quad P(3|2)$ $\quad P(\eta|2)$

$10 = 5 + ?$

Why did we use nature states?

Alternatively, we can define directly:

$\underline{p(y|x)}$ - prob. case

$\underline{Y(x) \subset Y}$ - nondet. case

$y = h(x, \eta)$

Time

$\sim$ = function of time
- time parameterized

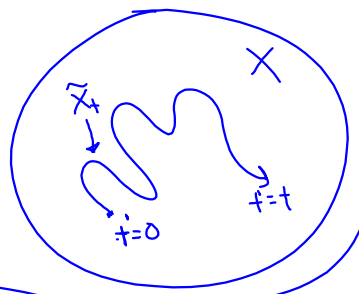
So far, we have seen only instantaneous observations

Let T be a time interval. $T = [0, \infty)$

Let $\tilde{x}_t$ denote a function $\tilde{x}_t : [0, t] \to X$

called the state history or

state trajectory



If we know $\tilde{x}_t$, then we have everything

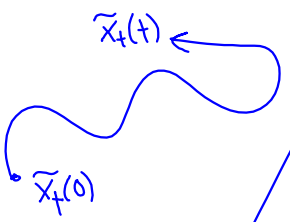
We can define a history-based sensor

For all $t \in T$, $y = h(\tilde{x}_t)$ $\quad h : \tilde{X} \to Y$

Ex Odometer

A point moves in $\mathbb{R}^2$

$X = \mathbb{R}^2 \quad T = [0, \infty)$



Odometer reports distance traveled

$Y = [0, \infty)$

If the point always moves at unit speed, then

$y = h(\tilde{x}_t) = t$

Ex Delayed state measurement

$$y = h(\tilde{x}_t) = \begin{cases} \tilde{x}_t(t-1) & \text{if } t \geq 1 \\ \tilde{x}_t(0) & \text{otherwise} \end{cases}$$

We can make a noisy (disturbed) history-based sensor

$$y = h(\tilde{x}_t, \nu)$$