Sensors with disturbances

\( \Psi \) : nature state space

\( \Omega = X \times \Psi \)

\( h: \Omega \rightarrow Y \)

\[ \Psi \] - we don't have direct access.

Cannot predict \( y \in \Psi \)

However, suppose \( y \) is observed and \( x \) is known.

Solve \( y = h(x,y) \) for \( y \).

Preimages are larger \( H(y) \) for all \( y \in Y \).

Forms a cover

\[ \bigcup_{y \in Y} H(y) = X \]
Probabilistic case
We are given $p(y)$ or $p(y|x)$
$p(y)$ or $p(y|x)$

Ex. A faulty (noisy) detection sensor
$V \preceq X$ is detection region
$Y = \{0, 1\}$, $\Psi = \{0, 1\}$

Given
\[ P(y=1|x \in V) \]
\[ P(y=0|x \notin V) \]
\[ P(y=1|x \notin V) \]
\[ P(y=0|x \in V) \]

$\gamma = y$
$\gamma = h(x,y)$
"Controlled experiment"

$H(0) = X$
$H(1) = X$

non-det.
Three cases

1) simple sensor \( y = h(x) \)

2) nondet.

3) probabilistic

\[ \Psi \]

Ex. Agent counter \( X = \mathbb{R}^2 \) \( Y = \{0, 1, \ldots, n\} \)

Ideally, \( h(x) = |A(x) \cap V| \)

\( \Psi = \{0, 1, 2\} \)

\[ X = h(x, \Psi) = |A(x) \cap V| + \Psi \]

Suppose \( P(y|x) \) is known.

\[ P(y|x) = P(y|x) \text{ for the unique } y \text{ such that } y = h(x) \]
$P(y|x) = \sum P(y|x) \text{ for the unique } y \text{ such that } y = h(x,y)$

$X \in Z \quad \mathbb{H} = \{0,1,2\}$

$Y = Z \quad P(1|2)$

$Y = x + y \quad P(3|2)$

$10 = 5 + \quad Y = h(x,y)$

Why did we use nature states?

Alternatively, we can define directly:

$p(y|x) - \text{ prob. case}$

$Y(x) < Y - \text{ nondet. case}$
Time

So far, we have seen only instantaneous observations.

Let $T$ be a time interval, $T = [0, \infty)$.

Let $\tilde{x}_t$ denote a function $\tilde{x}_t : [0, t] \rightarrow X$ called the state history or state trajectory.

We can define a history-based sensor $h$:

For all $t \in T$, $y = h(\tilde{x}_t)$

$h : \tilde{X} \rightarrow Y$

If we know $\tilde{x}_t$, then we have everything.
Example: Odometer
A point moves in \( \mathbb{R}^2 \)
\( X = \mathbb{R}^2, \quad T = [0, \infty) \)
\( x_0(0) \)
\( x_0(t) \)

If the point always moves at unit speed, then
\( Y = h(x_t) = t \)

Example: Delayed state measurement
\( Y = h(x_t) = \left\{ \begin{array}{ll}
\sum x_t(t-1) & \text{if } t \geq 1 \\
x_0(0) & \text{otherwise}
\end{array} \right. \)

We can make a noisy history-based sensor
\( Y = h(x_t, y) \)