Tracking an agent in a planar graph

- Agent moves continuously

\[ x = (x_1, x_2) \]
\[ X \subset \mathbb{R}^2 \]

\[ y = h(x) = x_1 \]
Tracking an agent in a planar graph

- \( e \rightarrow h \Rightarrow \text{impossible} \)
- \( a, b, c, d, f, \) or \( g \rightarrow h \Rightarrow \text{possible} \)
- \( h \rightarrow d \rightarrow a \)

Recall puzzle

\[
\begin{align*}
X & \quad Y & \quad h \rightarrow d \\
\text{a} \rightarrow \text{a} & \quad f \rightarrow \text{b} & \quad h \rightarrow h
\end{align*}
\]
For the last examples we reasoned about $\widetilde{X}_t(\tilde{Y}_t)$ by considering (propagating) each possible $X(\tilde{Y}_{t',t})$ for $t' \leq t$.

- Initially, we have $X(0)$ (or $X_0$).
- Then reason about $X(\tilde{Y}_{t',t})$ at some critical $t$'s.
Observation Histories Without a Perfect Clock

\( \gamma_t : [0, \infty) \rightarrow Y \) 
\[ \{(t, \gamma_t) : (t_2, \gamma_{t_2}), \ldots \} \]

- Change the "time stamps".
- Suppose we have an (internal) index set \( S \).
- The index set may be:
  1. Continuous: Usually \( S = [0, \infty) \subset \mathbb{R} \)
  2. Discrete: Usually \( S = \mathbb{N} = \{1, 2, 3, \ldots \} \)

As the sensor observations arrive, they are each "stamped" with some unique \( s \in S \).

Imagine pairs: \((\gamma, s)\)

- Stamp observation
- Define the observation history as \( \tilde{\gamma}_s : S(s) \rightarrow Y \) in which \( S(s) \subseteq S \) is the set of all indices up to and including \( s \).
Let \( \mathcal{Z} \) be the set of all monotonically increasing functions.

Instead of looking for some predefined set of codes, we can generalize the problem. Define a mapping \( \mathcal{Z} \to \mathcal{T} \), where \( \mathcal{T} \) is not given. Assume \( \mathcal{Z} \) is not given.

In any case, we assume \( \mathcal{Z} \) is an ordered set such that \( \forall x \in \mathcal{Z}, \forall y \in \mathcal{Z} \), \( x < y \Rightarrow f(x) < f(y) \).

How are we supposed to do it? What was the final result? Given \( \mathcal{Z} \), how could we have approached it?
The inference problem (assume all $\sigma \in \Sigma$ are invertible)

$$\tilde{X}(\tilde{Y}_s) = \left\{ \tilde{x}_t \in \tilde{X} \mid \exists \sigma \in \Sigma \text{ such that } \forall t' \in [0, 1], \tilde{y}_s(\sigma^{-1}(t')) = h(\tilde{x}_t(t')) \right\}$$

$$\sigma : S \rightarrow T$$

$$h : X \rightarrow Y$$

$$\tilde{X} = \bigcup_{t \in T} \tilde{X}_t$$

$$\tilde{Y} = \bigcup_{s \in S} \tilde{Y}_s$$

We have preimages in $\tilde{X}$, generated from each $\tilde{Y}_s$. 