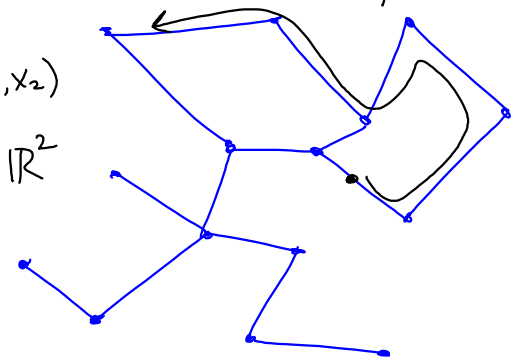
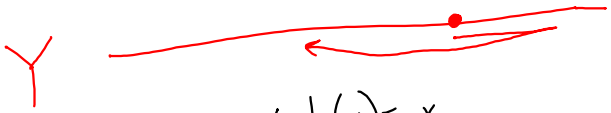
Tracking an agent in a planar graph

- Agent moves continuously

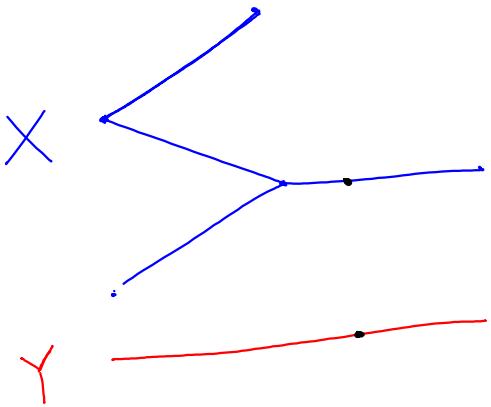
$x = (x_1, x_2)$

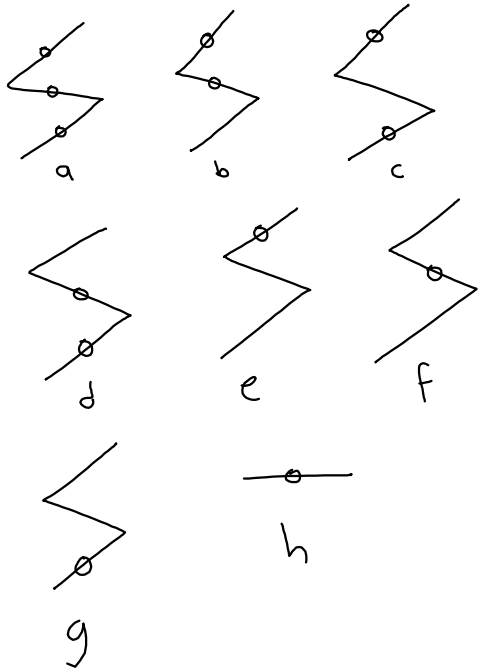
$X \subset \mathbb{R}^2$





$y = h(x) = x_1$





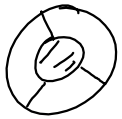
Tracking an agent in a planar graph

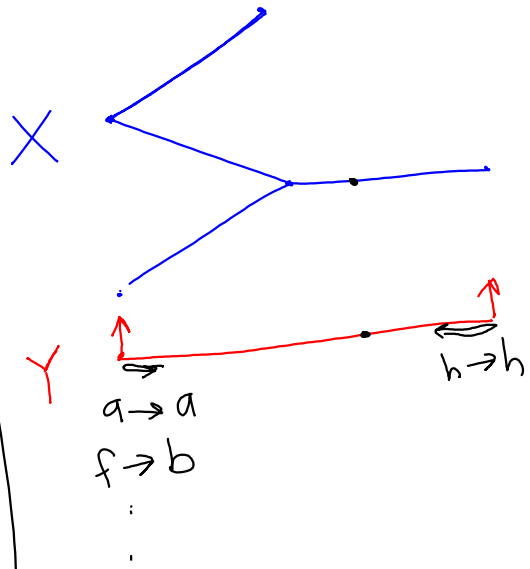
$h \to d \to a$

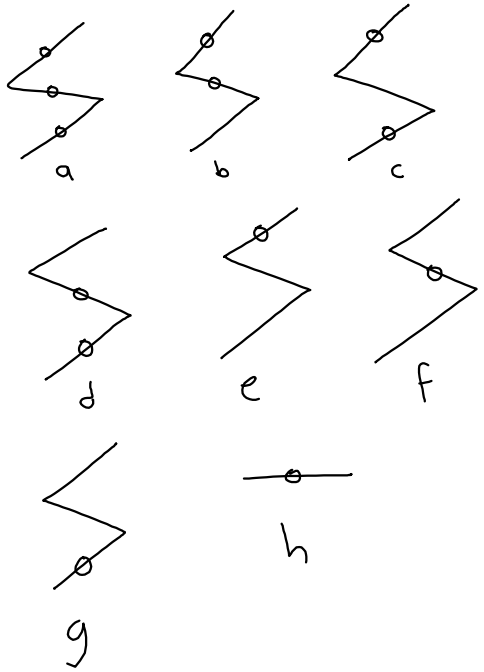
$e \to h \Rightarrow$ impossible.

$a, b, c, d, f,$ or $g \to h \Rightarrow$ possible

$h \to d$

recall puzzle 

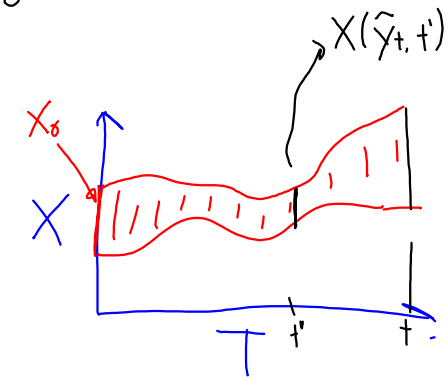




For the last examples we reasoned about $\tilde{X}_t(\tilde{y}_t)$ by considering (propagating) each possible $\underline{X(\tilde{y}_t, t')}$ for $t' \leq t$.

$X(\tilde{y}_t, t') \subseteq X$ → possible → subset of X at time t', given $\tilde{y}_t$

- Initially, we have $X(0)$ (or X_0)
- Then reason about $\underline{\underline{X(\tilde{y}_t, t')}}$ at some ~~critic~~ critical t''s



Observation Histories Without a Perfect Clock

$\tilde{Y}_t : [0,1] \to Y$ $\{(t_1, y_1), (t_2, y_2), \ldots\}$

change the "time stamps"

Suppose we have an (internal) index set S.

The index set may be:

1) continuous: Usually $S = [0, \infty) \subset \mathbb{R}$
2) discrete: Usually $S = \mathbb{N} = \{1, 2, 3, \ldots\}$

As the sensor observations arrive, they are each "stamped" with some unique $s \in S$

Imagine pairs: (y, s) stamp, observation

$S(s) = [0, s]$
or
$S(s) = \{1, 2, \ldots, s\}$

Define the observation history as

$\tilde{Y}_s : S(s) \to Y$ in which $S(s) \subseteq S$ is the set of all indices up to and including s.

We can generalize the inference problem:

Given $\tilde{y}_s$, what could $\tilde{x}_t$ have been?

what was the final t?

How are s and t related?

variable

In any case we always assume that

$$s' > s \Rightarrow t' > t$$

Define a mapping $\sigma : S \to T$, which relates s to t.

We assume σ is a monotonically increasing function

<u>Problem</u>: Often, we assume σ is not given.

Instead it belongs to some predetermined set Σ of functions

<u>Ex</u> Σ : set of all monotonically increasing functions

The inference problem (assume all $\sigma \in \Sigma$ are invertible (bijective))

$\sigma : S \to T$

$h : X \to Y$

$$\tilde{X}(\tilde{y}_s) = \{ \tilde{x}_t \in \tilde{X} \mid \exists \sigma \in \Sigma \text{ such that } \forall t' \in [0, t] \; \tilde{y}_s(\sigma^{-1}(t')) = h(\tilde{x}_t(t')) \}$$

$\sigma(s)$

$$\tilde{X} = \bigcup_{t \in T} \tilde{X}_t$$

$$\tilde{Y} = \bigcup_{s \in S} \tilde{Y}_s$$

We have preimages in $\tilde{X}$, generated from each $\tilde{y}_s$