A Short Survey on Pursuit-Evasion Games

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1 Introduction

Pursuit-evasion game is about how to guide one or a group of pursuers to catch one or a group of moving evaders. Because of its extensive applications, such as searching buildings for intruders, traffic control, military strategy, and surgical operation, a lot of research [1, 3, 4, 5, 6, 8, 7, 10, 9, 11, 12] has been done on this problem.

Pursuit-evasion games are also called mobile guards in art gallery problems, which are a variation of the art gallery problems [2]. Art gallery problems are originally about how to determine the minimum number of static guards to sufficiently guard an art gallery in the shape of polygons. Many variations are introduced after [2] was published. If static guards are considered, interesting variations include problems considering holes in the environment, limited view of guards, and three-dimensional art galleries. For problems with mobile guards, people are very interested in problems considering either known [3, 8, 7, 10] or unknown environment [5, 11, 12], guards with limited range and uncertain sensors [12], and differential motion models of players [6].

In this short survey, we will briefly summarize and discuss some paper [3, 6, 12] presented in the class on some recent progress on pursuit-evasion games. Several other references are also provided for people interested to do further investigation.

2 Problem Formulation

In the game theory, a game consists of three parts: players, actions and loss functions. The solutions to a game are normally policies for both players. With these policies, an equilibrium will be achieved and both players will have no regrets.

Two types of players in a pursuit-evasion game are the pursuer and the evader, respectively. Both types of players could be either one or a group. Normally, pursuers and evaders receive different information. In the worst case analysis [3, 8, 7], it is always assumed that evaders work as Nature, which know any information, such as the location of pursuers, the environment, and even the policy of pursuers. But pursuers do not know position of evaders. The pursuit-evasion game could be thought as a game of pursuers against Nature. In the probabilistic analysis [12], evaders are considered to be no superior than pursuers, which might only have limited range and uncertain sensors and not know the environment.

Actions to both players depend on the environment and the given motion model. The resulting state of an action should not collide with obstacles in the environment. If model of motion is discrete, actions are chosen from a finite set of either discrete inputs or states obtained by applying these discrete inputs. If a differentiable motion model is provided, actions are chosen from a function space, which consists of functions from time to input space. If a stochastic motion model is used, several states might be obtained by applying an input on a given state. Besides inputs, a probability distribution over the possible resulting states is also necessary to describe actions.

The loss of a player with a policy is the cost of the player executing the policy. The cost might be related to the execution time of the policy, consumed energy, or whether the evader is caught. Because the objectives of pursuers and evaders are opposite to each other and in most cases the loss of pursuer is the gain of the evader, the pursuer-evasion game could be thought as a zero-sum game. If uncertainty is modeled as function of some random variables, calculation of expected loss is necessary.

The objective of players in the pursuit-evasion game is to find policies which minimize their own loss. If execution time of the policy is used to evaluate the loss, pursuers is to catch evaders as soon as possible, and the evaders try to avoid being caught. Because actions of both players depends on each other, it might be impossible for each player to achieve its global minimum cost. People are more interested of policies at the equilibrium, which will reach the equilibrium and make both players feel no regret.

3 Methods to solve pursuit-evasion problems

3.1 Approach for Problems with Differential Motion Models [6]

If motion models of players in the pursuit-evasion game are differential equations, the game is also called differential games. The objective of game is to find the saddle-point equilibria. Obstacles normally do not exist in the environment to relieve the complexity of the problem. Typical examples of this kind of game include the Homicidal Chauffeur and the Lady in the Lake problem.

In the Homicidal Chauffeur game, a car which has a minimum turning radius tries to knock down a pedestrian, who try to dodge the car. The car can move faster but its turning radius is much bigger than the pedestrian. The objective is to design policies for both players. In the Lady in the Lake problem, a man runs around a circle and a woman moves from the circle center to the boundary of the circle. When the woman reaches the boundary, the distance between the man and the woman is measured by the angle between the segments connecting their position and the circle center. The man tries to minimize this angle, but the woman tries to maximize it. The objective of the game is to design policies for both players.

The key idea of solutions to differential games is to solve Isaacs equations, which is the necessary and sufficient condition for saddle-point equilibria.

3.2 Approach using Worst Case Analysis [3]

This type of approach assume that environment is known to pursuers and evaders work as Nature. The loss of pursuers is related to whether the evader is caught, so pursuers have to take policies based on worst-base analysis to avoid regret. Typical research along this direction includes [3, 7, 8, 9, 10]. We will only provide a brief summary of [3] which is presented in the class.

In [3], two main results are given. First, some bounds are provided for the minimum number of necessary pursers to detect evaders in some given polygonal environments. It is shown that $\Theta(\sqrt{h} + \lg n)$ pursuers are necessary for multiply-connected polygonal free spaces with n edges and h holes. For a simply-connected polygonal free space with n edges, $\Theta(\lg n)$ pursuers are necessary. Secondly, a complete algorithm is provided for a purser to detect an evader in a two-dimensional polygonal space. The basic idea of the method is to decompose the space based on critical events into conservative regions. Moving in a conservative region will not provide any new information to pursuers. Each conservative region corresponds to several information states, each of which represents the pursuers' current knowledge about pursuers, that is, whether pursuers are possibly in regions that border gap edges of current visibility polygon. Based on the space decomposition and information state, the pursuit-evasion problem is transformed into a search problem in the information graph, i.e., searching for a path from an information state, at which pursuers know nothing about evaders, to an information state, at which pursuers know for sure that there are no evaders in the environment.

3.3 Approach using Probabilistic Analysis [12]

When environment is unknown to the pursuer, one way is to first build the map and then solve the pursuit-evasion problem in a known environment. Another way is to use probabilistic pursuit-evasion game framework to solve both map building and pursuit-evasion game at the same time [12].

The probabilistic pursuit-evasion game framework is first pointed out in [4] to solve a game involving multiple pursuers and one randomly moving evader. In [11], the method is extended to solve a game with multiple pursuers and multiple randomly moving evaders. In [5], an eavder which actively avoids pursuers is considered in the above probabilistic framework and dynamic programming is used to find the Stackelberg equilibrium. The paper [12] provides a general overview and main idea of the research using the probabilistic method to solve the pursuit-evasion games.

The problem in [12] models a lot of practical considerations, which include the limited range and uncertainty of sensor, differential motion models of various players, unknown environments and exogenous disturbance to dynamics of players. To simplify the problem such that the problem could be solved in real time, a hierarchical hybrid system architecture is employed. Differential motion models of various players and exogenous disturbance to dynamics of players are dealt with in the lower level of the system such that the high-level strategy planner could possibly work in real time.

The basic idea behind the probabilistic pursuit-evasion game is based on information space and Bayesian reasoning. Information state in the paper is the sequence of observation of the position of pursuers, evaders and obstacles up to the current time. According to the observation and parameters of uncertainty of sensor (probability of false negative and false positive), pursuers could estimate the position of evaders and obstacles using Bayesian rules. Based on the estimation, two greedy algorithms are designed to provide pursuers policies to catch evaders. The local-max algorithm will guide pursuers to their neighbor states which have the highest probability to catch an evader. The global-max algorithm will guide pursuers one-step closer to states which have the highest probability over the whole state space to catch evaders.

4 Discussion and Conclusion

4.1 Information Space and State Space

When pursuers do not have perfect information about the current state, method in the information space is necessary. For pursuit-evasion games, the state of the game consists of the position of pursuers and evaders. Imperfect information might happens when the position of evaders, pursuers or both is unknown.

In the classical pursuit-evasion games, in which differential motion model is considered [6], the state of the game is known to both players because both the initial state and motion equation are given and states at any time can be described by algebraic equations. So, finding the solution to the problem is equivalently to solve the Isaacs equation. In [3, 12], the position of the evaders is unknown to pursuers and methods using information space are used to solve the problem. There is no mathematical formulation of the position of evaders at any time. It is impossible to find the solution by solving equations. Either worst case analysis or probabilistic analysis is necessary to solve the problem.

4.2 Worst Case Analysis and Probabilistic Analysis

Using the worst case analysis to find solutions for the pursuit-evasion game is equivalently to find the equilibrium of the game when the loss is only related to whether evaders are caught or not. When evaders are caught at the end of game, pursuers receive no loss; otherwise, pursuers will obtain a constant positive loss. In the worst case analysis, evaders normally play as Nature, we can assume that evaders always have perfect policies and will never feel regret even they are caught at the end of the game. The key of the game is how to design the pursuer's policy which will make pursuers feel no regret. To make sure that pursuers will not regret after the game, pursuers' policy has to consider the worst case of the movement of evaders. Since the position of evaders is unknown, it could be at any point in a set. The task of pursuers is to clear these sets, such as conservative regions in [3], in the space one by one until the whole space is searched.

Even though following policies from the worst case analysis for the pursuit-evasion game will not regret, the solution is very conservative since the velocities of evaders are assumed to be arbitrary fast and know the whole information of the game [3]. In practice, evaders might be no superior than pursuers, i.e., evaders also only have limited information about the state of the game and limited velocities [12]. In this case, instead of the worst case analysis, we could estimate the position of evaders from the history of observation and choose the best movement to minimize the expected loss according to the estimation.

4.3 Some thoughts on the Application of Game Theory

A theory normally provides a framework help people to solve a class of problems. By the application of game theory on a special problem, what we expect is to find a solution at the equilibrium point such that both players will not regret after the game. However, in [12], even though the game theory framework is employed to formulate the pursuit-evasion problem, the solution is not an equilibrium. One possible reason might be that calculating the solution equilibrium is too time consuming such that it is impossible to be implemented on the real-time system in the paper. Since equilibrium solutions are not pursued, the point of using game theory framework in [12] is not clear.

4.4 Remaining Questions

1. The worst case analysis for evaders with limited velocity [3].

In [3], the velocity of evader is assumed to be arbitrarily fast, which is not normally true. Because of limits of mechanical systems, there always exists some limit over the velocity of a moving system. With a given velocity upper bound, it is possible to provide less conservative and more practical solution than that in [3].

2. Consider the search cost [3].

In [3], the loss function is only related to whether the evader is caught or not such that it is possible a solution might consume much more resource than necessary. If search cost could be modeled in the pursuit-evasion game, then what is the solution with minimum cost?

3. Consider the uncertainty in GPS sensor [12].

The paper assumes that pursuers will know their own position according to the GPS. In practice, GPS can only guarantee that the player is a circle centered at the detected position with some given precision tolerance as radius. It is possible to model this uncertainty to provide a more robust solution.

4. Consider the localization without the GPS system [12].

The position of pursuers is provided by the GPS system, what if the GPS system is not available? The only information is the observation from pursuers. Both the position of pursuers and evaders have to be estimated by the current information state.

In this short survey, we briefly summarize and discuss some typical papers on recent research on pursuit-evasion games, which include methods using state space and information space. For methods using information space, the analysis could be either worst case or probabilistic. Some remaining questions are also provided.

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