1. Are the following languages regular? (Prove your answer):
   - \( L_1 = \{0^p1^q0^p+q \in \Sigma^* \mid p \geq 0, q \geq 0 \} \)
   - \( L_2 = \{0(n^2) \in \Sigma^* \mid n \geq 2 \} \)
   - \( L_3 = \{w \in \Sigma^* \mid w = xzx^R \text{ for } x, z \in \{0,1\}\{0,1\}^*\} \)

2. If \( L \) is a regular language, are the following languages regular? (Prove your answer):
   - \( \text{PER}(L) = \{x_1x_2x_3 \in \Sigma^* \mid x_3x_2x_1 \in L, x_1, x_2, x_3 \in \Sigma\Sigma^*\} \)
   - \( \text{EVEN}(L) = \{w \in \Sigma^* \mid x_1, x_2, \ldots x_{2k} \in \Sigma, \ x_1x_2x_3 x_4, \ldots x_{2k-1}x_{2k} \in L, \text{ and } w = x_2x_4x_6x_8 \ldots x_{2k}\} \)
   - \( \text{SQRT}(L) = \{x \in \Sigma^* \mid xy \in L, y \in \Sigma^*, \text{ and } |y| = |x|^2\} \)
   - \( \text{LOG}(L) = \{x \in \Sigma^* \mid xy \in L, y \in \Sigma^*, \text{ and } |y| = 2|x|\} \)

3. Let \( h \) be the homomorphism \( h(a) = 01, h(b) = 0 \).
   - Find \( h^{-1}(L_1) \), where \( L_1 = (10 \cup 1)^* \).
   - Find \( h(L_2) \), where \( L_2 = (a \cup b)^* \).
   - Find \( h^{-1}(L_3) \), where \( L_3 \) is the set of all strings of 0’s and 1’s with an equal number of 0’s and 1’s.

4. Construct a language that is not regular, and that satisfies the pumping lemma.

5. Find the minimum-state finite automaton equivalent to the transition diagram shown in Figure 1.

![Figure 1](image)

6. Determine the Myhill-Nerode equivalence classes for \( L = \{w \in \Sigma^* \mid w \text{ has an equal number of 0s and 1s}\}. \) Use your answer to show that \( L \) is not regular.
7. Consider the input string for a DFA. A useful abstraction is to think of the input string as written, from left to right, on a tape. In this abstraction, the DFA has a reader head, that reads the characters written on the tape. The reader head starts at the leftmost character written on the tape, and as it moves to the right, it reads the rest of the characters of the input string.

- A 2DFA is a DFA that is able to move the reader head to the left and to the right (not only to the right, as a DFA). Prove that every for every 2DFA, there is a standard DFA that recognizes the same language.
- A one-pebble 2DFA is a 2DFA with the added capability of marking a character on the tape by placing a pebble on it. The transition equation depends on the present state, the tape character scanned, and the presence or absence of a pebble on the tape squared scanned. The output of the transition equation indicates the next state, a direction of motion for the reader head, and possibly placing or removing the pebble from the scanned character. The automaton rejects a string if it attempts to place a second pebble on the tape. Prove that for every one-pebble 2DFA, there is a DFA that recognizes the same language.

8. [Extra for fun and no credit] A robot is scouting in a 2-dimensional grid. The universe, as seen by the robot, consist of a 2-dimensional array of cells. The robot occupies one of the cells at any given instant. Some of the cells are “land cells”, and others are “water cells”. The robot can “see” its eight surrounding cells, as well as the cell it currently occupies. It then uses this local information and its current state (which is one of finitely many states) to compute its next state and next move. The available moves are go north, go south, go east, go west, stay. The robot can move only to a cell in its immediate neighborhood. The robot can transverse land and water cells.

For the following parts, a high-level solution that does not get into grungy nuts-and-bolts type of implementation details is acceptable; however it must be precise and unambiguous.

- Assume that the grid world consist of a finite rectangular landmass that contains a single lake in it. The boundaries of the landmass lie along the horizontal and vertical rows of the grid world. Prove that the robot can recognize whether the lake is also a rectangle with sides parallel to the sides of the landmass, once it is placed in the landmass.
- When placed in the same landmass of the previous part, can the robot identify whether the lake is a square with sides parallel to the sides of the landmass?
- Assume the landmass is infinitely large, and does not have water bodies in it. One of the cells contains a powerpack required to recharge the robot’s batteries. Is there a DFA (robot) such that regardless of its initial location and the location of the powerpack, the robot can find the powerpack? Prove your answer.