Ten Simple Rules
For Mathematical Writing

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ON WRITING

• “Easy reading is damn hard writing” (Hawthorne)

• “Word-smithing is a much greater percentage of what I am supposed to be doing in life than I would ever have thought” (Knuth)

• “I think I can tell someone how to write but I can’t think who would want to listen” (Halmos)
WHAT IS MATH WRITING?

• Writing where mathematics is used as the primary means for deduction and problem solving.

• Examples that are:
  – Math papers and textbooks
  – Analysis of mathematical models in engineering, physics, economics, finance, etc

• Examples that are not:
  – Novels, essays, letters, etc
  – Experimental papers and reports
WHAT IS DIFFERENT ABOUT MATH WRITING?

- Math writing blends two languages (natural and math)
  - Natural language is rich and allows for ambiguity
  - Math language is terse and must be unambiguous

- Math writing requires slow reading
  - Often must be read several times
  - Often is used as reference
  - Usually must be read selectively and in pieces
WHY THIS TALK?

- Experience is something you get only after you need it...
- **Current model: The conversational style**
  - “Mathematics should be written so that it reads like a conversation between two mathematicians on a walk in the woods” (Halmos)
  - “Talk to your readers as you write” (Strang)
  - Very hard to teach to others (“Effective exposition is not a teachable art. There is no useful recipe …” Halmos)
  - Controversial (where do proofs start and end? … I am not sure what the assumptions are … I can’t find what I need … etc)

- **Instead we will advocate a structured style**
  - Offers specific verifiable rules that students can follow and thesis advisors can check
  - Allows room to develop and improve over time
SOURCES

• General style books
  – Strunk and White, “The Elements of Style” (www)
  – Fowler and Aaron, “The Little Brown Handbook”

• Halmos, “How to Write Mathematics”

• Knuth, et al, “Mathematical Writing” (www)

• Kleiman, “Writing a Math Phase Two Paper,” MIT (www)

• Krantz, “A Primer of Mathematical Writing”

• Higham, “Handbook of Writing for the Mathematical Sciences”

• Thomson, “A Guide for the Young Economist”
RULES OF THE GAME

• Small rules:
  – Apply to a single sentence (e.g., sentence structure rules, mathspeak rules, comma rules, etc)

• Broad rules:
  – Apply to the entire document
  – General style and writing strategy rules
  – Are non-verifiable (e.g., organize, be clear and concise, etc)

• Composition rules (our focus):
  – Relate to how parts of the document connect
  – Apply to multiple sentences
  – Are verifiable
EXAMPLES OF SMALL RULES I

• Break up long sentences into simple ones
• Use active voice ("we" is better than "one")
• Mathspeak should be "readable"
  – BAD: Let $k>0$ be an integer.
  – GOOD: Let $k$ be a positive integer.
  – BAD: Let $x \in \mathbb{R}^n$ be a vector.
  – GOOD: Let $x$ be a vector in $\mathbb{R}^n$.

• Don’t start a sentence with mathspeak
  – BAD: Proposition: $f$ is continuous.
  – GOOD: Proposition: The function $f$ is continuous.
EXAMPLES OF SMALL RULES II

• Minimize “strange” symbols within text
• Do no repeat words too closely
• Minimize the use of “very,” “trivial,” “easy,” “nice,” “fundamental,” etc
• Use abbreviations correctly (e.g., cf., i.e., etc)
• Comma rules
• “Which” and “that” rules
• ... ETC
EXAMPLES OF BROAD RULES

- Language rules/goals: precision, clarity, familiarity, forthrightness, conciseness, fluidity
- Organizational rules (how to structure your work, how to edit, rewrite, proofread, etc)
- “Down with the irrelevant and the trivial” (Halmos)
- “Honesty is the best policy” (Halmos)
- “Defend your style” (against copyeditors - Halmos)
- ... ETC
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TEN COMPOSITION RULES

• Structure rules (break it down into digestible pieces)
  – Organize in segments
  – Write segments linearly
  – Consider a hierarchical development

• Consistency rules (be boring creatively)
  – Use consistent notation
  – State results consistently
  – Don’t underexplain - don’t overexplain

• Readability (make it easy for the reader)
  – Tell them what you’ll tell them
  – Use suggestive references
  – Consider examples and counterexamples
  – Use visualization when possible
ORGANIZE IN SEGMENTS

• “Composition is the strongest way of seeing” (Weston)

• Question: What is the fundamental unit of composition?

• Segment: An entity intended to be read comfortably from beginning to end

• Examples:
  – A result and its proof
  – An example
  – Several results/examples with discussion

• A segment should “stand alone” (identifiable start and end, transition material)

• Length: 1/2 page to 2-3 pages
EXAMPLE: A SECTION ON PROBABILITY MODELS

- Sample space - Events (1 page)
- Choosing a sample space (0.5 page)
- Sequential models (0.75 page)
- Probability laws - Axioms (1.25 page)
- Discrete models (2 pages)
- Continuous models (1 page)
- Properties of probability laws (2 pages)
- Models and reality (1.25 page)
- History of probability (1 page)
WRITE SEGMENTS LINEARLY

- Question: What is a good way to order the flow of deduction and dependency?
- General rule: Arguments should be placed close to where they are used (minimize thinking strain)
- Similarly, definitions, lemmas, etc, should be placed close to where they are used
- View ordering as an optimization problem
EXAMPLE

Dependency Graph of Arguments

Level 1 Arguments

Level 2 Arguments

Nonlinear

Linear

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CONSIDER A HIERARCHICAL DEVELOPMENT

- Arguments/results used repeatedly may be placed in special segments for efficiency

- Possibly create special segments for special material (e.g., math background, notation, etc)

- Analogy to subroutines in computer programs
USE CONSISTENT NOTATION

• Choose a notational style and stick with it
• Examples:
  – Use capitals for random variables, lower case for values
  – Use subscripts for sequences, superscripts for components
• Use suggestive/mnemonic notation. Examples: S for set, f for function, B for ball, etc
• Use simple notation. Example: Try to avoid parenthesized indexes: $x(m,n)$ vs $x_{mn}$
• Avoid unnecessary notation:
  – BAD: Let $X$ be a compact subset of a space $Y$. If $f$ is a continuous real-valued function over $X$, it attains a minimum over $X$.
  – GOOD: A continuous real-valued function attains a minimum over a compact set.
STATE RESULTS CONSISTENTLY

• Keep your language/format simple and consistent (even boring)
• Keep distractions to a minimum; make the interesting content stand out
• Use similar format in similar situations
• Bad example:
  – Proposition 1: If A and B hold, then C and D hold.
  – Proposition 2: C’ and D’ hold, assuming that A’ and B’ are true.
• Good example:
  – Proposition 1: If A and B hold, then C and D hold.
  – Proposition 2: If A’ and B’ hold, then C’ and D’ hold.
DON’T OVEREXPLAIN - DON’T UNDEREXPLAIN

• Choose a target audience level of expertise/background (e.g., undergraduate, 1st year graduate, research specialist, etc)
• Aim your math to that level; don’t go much over or under
• Explain potentially unfamiliar material
• Consider the use of appendixes for background or difficult/specialized material
**TELL THEM WHAT YOU’LL TELL THEM**

- **Keep the reader informed** about where you are and where you are going
- Start each segment with a short introduction and perhaps a road map
- Don’t string together seemingly aimless statements and surprise the reader with “we have thus proved so and so”
- Announce your intentions/results, e.g., “It turns out that so-and-so is true. To see this, note …”
- Tell them what you told them
USE SUGGESTIVE REFERENCES

• Frequent numbered equation/proposition references are a **cardinal sin**
• Page flipping wastes the reader’s time and breaks concentration
• Refer to equations/results/assumptions by content as much as possible
• Repeat simple math expressions
• Remind the reader of unusual notation, and earlier analysis
• Dare to be redundant (but don’t overdo it)
CONSIDER EXAMPLES AND COUNTEREXAMPLES

• “Even a simple example will get three-quarters of an idea across” (Ullman)
• Examples should have some spark, i.e., aim at something the reader may have missed
• Illustrate definitions/results with examples that clarify the boundaries of applicability
• Use counterexamples to clarify the limitations of the analysis, and the need for the assumptions
USE VISUALIZATION WHEN POSSIBLE

• “A picture is worth a thousand words”
• Keep figures simple and uncluttered
• Use substantial captions
• Captions should reinforce and augment the text, not repeat it
• Use a figure to illustrate the main idea of a proof/argument with no constraint of math formality
• Prefer graphs over tables
“Bad thinking never produces good writing”

(Lamport)

Good writing promotes good thinking …